

**Summary of Some Contributions of Lawrence P. Horwitz,**  
Recipient of 2008 TGA Award

The following summarizes some of the major contributions that have emerged from the publications and studies of Professor Horwitz:

1. Hypercomplex agebras in quantum theory

It was known to von Neumann that the formulation of the quantum theory does not require the restriction to a complex number field, but that the use of the non-commutative (skew) field of quaternions also works. It was suggested by von Neumann to his colleague Herman Goldstine that an even greater generalization is possible, to the non-associative number system of Cayley numbers, or octonions, which is the last division algebra over the reals. Hamilton had investigated both of these number systems. Goldstine and Horwitz found that it is possible to build a Hilbert space over the octonions [H.H. Goldstine and L.P. Horwitz, Proc. Nat. Aca. **48**, 1134 (1962)]. Some consequences of this work were explored in [L.P. Horwitz and L.C. Biedenharn, Jour. Math. Phys. **20**, 269 (1979)]. Horwitz then returned to develop a full theory of quaternionic quantum mechanics, later discussed in a book of Stephen L. Adler (Institute for Advanced Study in Princeton) called "Quaternionic Quantum Mechanics and Quantum Fields", Oxford University Press (1995), which was very much a joint effort over a period of years during visits to IAS, largely motivated by Horwitz's work, but with significantly more content consisting of general results as well as examples of possible physical applications.

In this context, Adler found a way to extract variations of Hamilton functions and Lagrangians that could be used to define a (non-relativistic) quantum field theory, and then discovered, with a result shown by his student, A.C. Millard at Princeton, a form of statistical mechanics in which, in the ensemble average, ordinary quantum mechanics could emerge. This resulted in a second book by Adler, "Quantum Theory as an Emergent Phenomenon", Cambridge University Press (2004), which also involved some significant collaboration with Horwitz.

The underlying physics of the relatively new methods of stochastic analysis introduced by L. Diosi and N. Gisin, and eventually L.P. Hughston, G.C. Ghirardi, P. Pearle, A. Rimini, Horwitz and Adler, and others, for the collapse of quantum wave functions, can be modelled in this framework. Horwitz has been involved in considerable efforts in this direction, both in applications, and also searching for the rather deep goal of making a connection with the theory of semigroups discussed below.

2. Algebraic approach to the quark model

In 1964, there were many expositions on the "quark model" of hadronic physics at CERN, and Horwitz (then resident at University of Geneva) brought the question to Yuval Ne'eman whether these results could be explained in term of group theory rather than the very questionable dynamics (at that time) of such strongly interacting systems. They succeeded (with N. Cabibbo) in developing a group theoretical model which was very

successful, and later justified its structure in terms of the asymptotic forms proposed by Gell-Mann and his student Melosh. This work attracted much attention by phenomenologists, but eventually gave way to studies in non-Abelian quantum field theory, particularly after the seminal work of 't Hooft and Veltman on renormalization of gauge fields and Gross and Politzer on asymptotic freedom.

### 3. Unstable Systems

In the years 1964-1971 on visits to CERN and the University of Geneva, Horwitz's interest turned to the description of unstable systems, and with J.-P. Marchand, he set the mathematical foundations of what may be called the "decay-scattering" system, within the general framework of the ideas of Wigner and Weisskopf. A resolvent formalism was developed for treating this theory, but it became clear as a result of experiments performed at Fermilab that this model was inadequate for the description of more than one channel decay, such as the neutral K meson system. Horwitz and collaborators showed that on the soluble Lee-Friedrichs model, the errors implicit in the Wigner-Weisskopf model were beyond the experimental uncertainties. It became clear that there was a need for the realization of an exact semigroup (not realized in the Wigner-Weisskopf model) for the evolution of an unstable system (as in the phenomenological model of Yang, Oehme and Wu), which resulted in his effort to bring the theory of Lax and Phillips [P.D. Lax and R.S. Phillips, "Scattering Theory," Academic Press. N.Y. (1967)], developed for classical acoustic or electromagnetic waves into the framework of the quantum theory.

### 4. The quantum Lax-Phillips theory

Horwitz's former doctoral student, Yossi Strauss, at Tel Aviv University, found a way to apply the Lax-Phillips theory to the quantum mechanical case, resulting in a long effort to develop the theory of semigroups for its application to unstable systems. New results have been continuously found along the way. A considerable number of workers in the field have appreciated these developments, and there is a growing literature including the important recent discovery (by Strauss) of a self-adjoint operator (to be published) for which the expectation value, in the framework of standard nonrelativistic quantum theory, decreases monotonically. This operator, which provides an important link to irreversible processes, is being studied for its consequences by Horwitz, Strauss and a group of doctoral students at Tel Aviv.

### 5. Relativistic quantum theory

A relativistic formulation of the description of unstable particle decay is necessary for a quantum mechanical description (which is not quantum field theoretical, in order to obtain local information and interference effects in a simple way) since the particles in decay products undergo changes in mass. Horwitz therefore became interested in developing a relativistic classical and quantum dynamics in 1971, with his colleague C. Piron, who also realized that no such consistent theory existed (even though there had been much literature by Bergmann, Picasso, Synge, and others), and they developed a good theory which coincided with the original work of Stueckelberg in 1941, but was more general in an essential way. Stueckelberg, and later Feynman and Schwinger had considered one-particle

theories, but the Horwitz-Piron theory [L.P. Horwitz and C. Piron, *Helv. Phys. Acta* **46**, 316 (1973)], with an invariant universal time parameter (which can be identified with the Newtonian time) allowed the construction of many-body theories. They, with some colleagues, worked out the manifestly covariant Gibbs ensembles and Boltzmann theories.

In this theory, the time  $t$  of Einstein, which varies covariantly with the space coordinates  $x$  under Lorentz transformation due to a change in frame of the observer's apparatus, is considered a dynamical variable. This interpretation is consistent with Einstein's thought experiment in which there are two frames, say,  $F$  and  $F'$  moving at constant velocity relative to one another. Signals emitted in  $F$  are then detected in  $F'$  with a different interval between them, consistent with the Lorentz transformation, and thus the time of arrival of a signal in  $F'$  is affected by the motion of the frame. In order to observe that the time of arrival of a signal is different than the time of emission in  $F$ , it is necessary that there be a common basis for detecting such times, as for a standard ruler for measuring the difference in apparent lengths induced by the Lorentz contraction. One comes to the conclusion that there must be a universal time (sometimes referred to in the texts, e.g., Taylor's book, as clocks made "in the same factory"). The Stueckelberg-Horwitz-Piron universal time  $\tau$  is this basic time, which in the above example can be thought of as the time of emission of a signal, and the Einstein time, entering on a dynamical level, as the time detected in the frame  $F'$ . The theory therefore has explicitly a universal time that can be identified with Newton's time, a postulate of the theory. In the classical theory, there is therefore (for an  $N$  body system) an  $8N$  dimensional phase space, with Hamilton equations on space and the (Einstein) time, parametrized by  $\tau$ , and in the quantum theory, wave functions on space and (Einstein) time, parametrized by  $\tau$  and evolving according to a Stueckelberg-Schrödinger equation where  $\tau$  plays the role of the Schrödinger nonrelativistic  $t$ . The usual Maxwell equations provide fields that are functions of the Einstein  $t$ , and transform covariantly. However, introducing gauge functions into the Stueckelberg-Schrödinger theory, requires the introduction of a fifth potential function, i.e., a 5D electrodynamics. Horwitz and collaborators have shown that there is a possibility that the nonlinear Lorentz-Dirac equations associated with this 5D electrodynamics induces a chaotic motion of the classical relativistic charged particle that may, on the average, stabilize its motion, and solve the well-known problem of the instability of the Abraham-Lorentz-Dirac equation. This question is under active investigation,

Horwitz and his doctoral student, R. Arshansky, were able to understand the famous Newton-Wigner localization question, and the Landau-Peierls construction in this framework. They also solved completely the quantum relativistic Kepler (Coulomb potential) problem in a manifestly covariant form, and found results coinciding with the Schrödinger spectrum. Its consequences for calculations such as the anomalous moment of the electron, self-interaction, and other properties of relativistic dynamical systems are under study.

In 1976 Horwitz and Yitzhak Rabin (now department chairman at Bar Ilan University) predicted, due to the coherence of the covariant wave function in space and time, an interference effect in time. To see this effect, due to the smallness of Planck's constant and the largeness of the velocity of light, frequencies of the order of a million million Herz or higher had to be used. An experiment was done at Max Planck Institute by Lindner *et al* about a year and a half ago, and the interference effect was found. This result is consistent

with the interpretation, pointed out above, of the Einstein time as a dynamical variable.

To understand the effect, it may suffice to describe the experiment. One and a half waves of laser light (about 850 nm) was directed toward Argon gas. Two positive pulses, one after the other, could cause an electron to be emitted at an earlier or later time. Just as in the space interference effect, where an electron may pass through one slit or another, the two possibilities of emission interfered with each other, and instead of seeing two pulses at the detector, there was a wave of interference over time, a time signal (like music). One of the experimentalists used the standard non-relativistic Schrödinger equation to account for the result. It is easy to show that numerically, one can find a reasonable approximation, but the calculation appears to be not correct. The nonrelativistic theory cannot have coherence in time in quantum theory (e.g., the books of G. Ludwig, “Foundations of Quantum Mechanics I, II” Springer-Verlag, New York, (1983, 1985). Horwitz [L. Horwitz, Phys. Lett. A **355** 1, (2006)] explained this point, and why even numerically, without regard to the interdiction on this construction, the result could only be approximate and becomes worse as one look at further away components of the pattern. In this paper, he calculated the result using the covariant theory, and in a very simple way, found a result that agreed with the experiment.

This result appears to be an important verification of the covariant form of the quantum theory. As the experimentalists asserted, this new technique makes possible new probes into the structure of matter, both for chemical structure as well as the physical possibility of new methods of communication.

## 6. Hamiltonian chaos

In a somewhat different direction, Horwitz and collaborators [Lawrence Horwitz, Y. Ben Zion, M. Lewkowicz, M. Schiffer and J. Levitan, Phys. Rev. Lett. **98** 234301 (2007)] found a way to imbed classical nonrelativistic Hamiltonian motion (with potential model for interaction) into a Riemannian geometrical framework. This was done by using an equivalence with an (auxiliary) Hamiltonian with conformal metric, inducing directly a Riemannian geometry, and then transforming back to a covariant form of the Hamilton orbits. The connection form induced under this transformation has torsion, but the torsion term cancels out along geodesics. [It is possible that the motion of a tensor particle (such as a particle with spin) would be sensitive to this torsion effect, and these ideas are under investigation.] The resulting calculation of geodesic instability has proven to be remarkably effective in predicting chaotic behavior for a wide class of potential models. The use of the conformal embedding is currently under investigation, by Horwitz and collaborators, in applications in general relativity, such as Bekenstein’s approach to Milgrom’s modification of Newtonian dynamics.