Concerning the Mistakes Committed by F. W. Hehl, C. Kiefer, et al. on Black Holes, Expansion of the Universe, and Big Bangs.

In his email postscripts, Professor Dr. F. W. Hehl provides the link http://www.thp.uni-koeln.de/gravitation to the Gravitation and Relativity group at the University of Cologne, wherein it is stated that this organisation studies black holes and relativistic cosmology, amongst other things. There is there a link to a monograph entitled *Thermodynamics of Black Holes and Hawking Radiation* (TBH), by one Prof. Dr. Claus Kiefer. In that monograph Kiefer cites a book entitled *Black Holes: Theory and Observation*, by Hehl, Kiefer and Metzler. There is also on the aforementioned website a link to a paper entitled *Emergence of Classicality for Primordial Fluctuations: Concepts and Analogies* (ECPF), by Kiefer and Polarski, which deals with certain aspects of Big Bang cosmology. In view of the foregoing I take it that what is expounded in these writings are the general views of Hehl as well, on black holes and cosmology.

On Black Holes

In TBH there is given some history of black holes. That history is incomplete, selective, shallow, and inaccurate. An accurate and fully documented history of black holes is given in *A Brief History of Black Holes*, at www.geocities.com/theometria/holes.pdf, so I will not reiterate the history here. In reference to the latter document the reader will find that Kiefer's history is clearly misleading and mostly erroneous. However, I will say that what he calls the "Schwarzschild" solution is not Schwarzschild's 1915 solution at all, and that one cannot get a black hole from Schwarzschild's true solution. The "solution" actually referred to by Kiefer, adduced in his equation 1.3 on page 6 of TBH, is that due to David Hilbert (December, 1916), which is a corruption of Schwarzschild's (1915) solution and also of that solution obtained by Johannes Droste (May 1916).

In equation 1.1 on page 4 of TBH the variable $r \equiv R_0$ is called, "Schwarzschild's radius" by the Author. In the context of equation (1.1) this is misleading. By calling this variable the "Schwarzschild's radius" it is plain that Kiefer et al. consider r to be the radius not only in relation to the Michelle-Laplace dark body (which is not a black hole) of Newton's theory, but also in relation to Einstein's gravitational field, notwithstanding this particular value $R_0 = 2Gm/c^2$. However, Newton's theory is formulated in Efcleethean[1] 3-space, whereas Einstein's theory is formulated in terms of a pseudo-Riemannian space. Consequently, the said "radius" cannot have the same meaning in Einstein's gravitational field as it does in Newton's theory. Now concerning his equation 1.3, Kiefer says

> *"One easily recognises the singularities in the metric (1.3) at $r = 0$ and $r = 2GM = R_0$. While the singularity at $r = 0$ is a real one (divergence of curvature invariants), the singularity at $r = R_0$ is a coordinate singularity ..."*

Now I remark that it is clear that this Author and his colleagues have already made several unsubstantiated assumptions. As with all orthodox relativists, they obtain a line-element that satisfies the field equations, but that is all. They simply inspect their line-element and assume that their quantity "r" is the radius therein, that there is only one radius and that their "radius" r can go down to zero. Then they additionally assume that Einstein's gravitational field requires of necessity that a singularity can occur and that this singularity must only occur where the Riemann tensor scalar curvature invariant (the Kretschmann scalar), $f = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$ is unbounded. Then they go looking for a "transformation of coordinates" that satisfies all their assumptions on the "radius" and on the Kretschmann scalar, find one in the fanciful Kruskal-Szekeres coordinates, and claim the existence thereby of the black hole. The Kruskal-Szekeres "extension" is not an extension. It describes a separate pseudo-Riemannian manifold that has nothing to do with Einstein's gravitational field. It does not give a coordinate patch for a section of the gravitational manifold that is not otherwise covered. The orthodox relativists merely leap between two disjoint manifolds by their "transformation of coordinates", but think that they move between coordinate patches on a single manifold. That is utter nonsense. Consequently, the development from equation (1.4) through to equation (1.15) in TBH is, although standard, entirely erroneous. Instead of wading through each and every falsity in TBH, I shall deal now with the fundamental geometrical issues which invalidate the whole of the monograph under consideration.

Everything must be determined by the line-element, not by foisting arbitrary assumptions upon the line-element and the components of the metric tensor. The line-element contains the full description of the intrinsic geometry of the space it represents.

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[1]For the geometry due to Efcleethees, usually and abominably rendered Euclid.

Consider any line element of the form

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

$$A(r), B(r), C(r) > 0,$$

where r is a quantity related to radial distance in Minkowski space. (Note: the inequality is required in the case of General Relativity on account of the necessity of Lorentz signature.) This line-element is a generalisation of the Minkowski line-element,

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

$$r \geq 0. \tag{2}$$

Conversely, the Minkowski line-element is a particular case of the general expression above (set $A(r) = B(r) = 1$, $C(r) = r^2$, say).

The solution to $R_{\mu\nu} = 0$ in terms of expression (1) is (using G = c = 1)

$$ds^2 = \left(1 - \frac{2m}{\sqrt{C(r)}}\right)dt^2 - \left(1 - \frac{2m}{\sqrt{C(r)}}\right)^{-1} d\sqrt{C(r)}^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$2m < \sqrt{C(r)} < \infty.$$

Equations (1) and (2) (and hence the above line-element for the gravitational field) share the same fundemantal geometry as they are line-elements of the same form. In line-elements of the form (1), the radius of curvature R_c is always the square root of the negative of the coefficient of the infinitesimal angular terms and the proper radius R_p is always the integral of the square root of the negative of the term containing the square of the differential element of the radius of curvature, which are purely geometrical issues not open to hypothesis or opinion. This is disguised in the line-element (1) above, so I write that line-element as

$$ds^2 = A(R_c(r))dt^2 - B(R_c(r))dR_c^2(r) - R_c^2(r)(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$A(R_c(r)), B(R_c(r)), R_c(r) > 0. \tag{3}$$

Then

$$R_c = R_c(r) = \sqrt{C(r)},$$

$$R_p = R_p(r) = \int \sqrt{B(R_c(r))}\, dR_c(r).$$

It is plain that the proper radius and the radius of curvature are not in general the same, and are not in general the same as the quantity "r" associated with Minkowski space. But in Minkowski space the proper and curvature radii are identical. This is a direct result of the fact that Einstein's gravitational field is pseudo-Riemannian, not pseudo-Efcleethean, but Minkowski space is pseudo-Efcleethean.

The fundamental issue is this: given a distance D between a point-mass and a test particle in pseudo-Efcleethean Minkowski space, what is the corresponding distance in the gravitational field? The answer is obtained by a mapping of the distance D in Minkowski space into a corresponding distance in the gravitational field. However, Einstein's gravitational field is pseudo-Riemannian. A peculiarity of this geometry is that the mapping of D is not one-to-one, but one-to-two. The parametric distance D is mapped into the radius of curvature $R_c(D)$ and the proper radius $R_p(D)$, which are determined entirely by the intrinsic geometry of the gravitational line element, because *a geometry is completely determined by the form of the line-element describing it.*

Recall the distance formula in the plane,

$$D^2 = (x_1 - x_0)^2 + (y_1 - y_0)^2,$$

which gives the distance between *any* two points in the plane. If $x_0 = y_0 = 0$ this obviously reduces to

$$D^2 = x_1^2 + y_1^2,$$

which determines a distance from the origin of the coordinate system.

If D and (x_0, y_0) are fixed and x and y are allowed to vary, one obtains

$$(x - x_0)^2 + (y - y_0)^2 = D^2,$$

which describes a circle, centre (x_0, y_0), radius D. If $x_0 = y_0 = 0$, this reduces to

$$x^2 + y^2 = D^2,$$

which still describes a circle of radius D, but now centred at $(0, 0)$. The location of the centre of the circle is immaterial for the study of the intrinsic geometry of the circle. The centre of the circle does not need to be located at the origin of the coordinate system.

Similarly, in Efcleethean 3-Space, if D and (x_0, y_0, z_0) are fixed,

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = D^2,$$

describes a sphere of radius D centred at (x_0, y_0, z_0). If $x_0 = y_0 = z_0 = 0$, this reduces to

$$x^2 + y^2 + z^2 = D^2,$$

which still describes a sphere of radius D, but now centred at $(0, 0, 0)$. The location of the centre of the sphere is immaterial for the study of the intrinsic geometry of the sphere. One does not need to locate the centre of the sphere at the origin of the coordinate system, and this, in combination with the intrinsic geometry of the line-element, is the simple crux of the whole matter of the confusion of the relativists about horizons and singularities, black holes and big bangs in Einstein's gravitational field.

Let D be the variable distance in Minkowski space between the point-mass and the test particle. Thus D is a parametric distance and Minkowski space a parametric space for the gravitational field. The point-mass does not need to be located at the origin of coordinates for Minkowski space; it can be located anywhere in Minkowski space, just as a sphere can be located anywhere in Efcleethean 3-Space and a circle anywhere in the Efcleethean plane. Consider now a radial line, infinitely extended in both directions, through the origin of the coordinates in Minkowski space (i.e. $r = 0$). This constitutes the real line. The direction of this radial line is immaterial. Also, for the purposes of our problem statement we are only interested in radial motion. Therefore, let the point-mass be located at a fixed *point* anywhere on the radial line, at say $r_0 \neq 0$. This denotes a *point* on the radial line, just as the origin of coordinates at $r = 0$ denotes a *point* on the radial line (indeed, just points on the real line). Let the test particle be located on the same radial line, at some variable distance r. This also denotes a *point*: a radially moving point. The distance between point-mass and test particle is

$$D = |r - r_0|, \tag{5}$$

because r may be above or below r_0 on the radial line, i.e. it may be at a greater or smaller distance from the origin of the coordinate system for Minkowski space than is the fixed point-mass. As $r \to r_0^{\pm}$, $D \to 0$, as given by equation (5), *irrespective of the value assigned to* r_0: r_0 is entirely arbitrary. Now Minkowski space is a parametric space for the gravitational field and so D is a parametric distance for the proper radius $R_p(D)$ and for the radius of curvature $R_c(D)$, which are the corresponding distances in the gravitational field. Recall that the radius of curvature is the square root of the negative of the coefficient of the infinitesimal angular terms of the line element and the proper radius is the integral of the square root of the negative of the term containing the square of the differential element of the radius of curvature. Now I have deduced in my published papers that

$$R_c = R_c(D) = \sqrt{C(D)} = (D^n + \alpha^n)^{\frac{1}{n}},$$

$$R_p = R_p(D) = \sqrt{R_c(D)\,(R_c(D) - \alpha)} + \alpha \ln \left| \frac{\sqrt{R_c(D)} + \sqrt{R_c(D) - \alpha}}{\sqrt{\alpha}} \right|,$$

$$\alpha = 2m, \quad n \in \Re^+, \quad D \in \Re^+, \tag{6}$$

where n is an arbitrary constant. But $D = |r - r_0|$, so

$$R_c(D) \equiv R_c(r) = (|r - r_0|^n + \alpha^n)^{\frac{1}{n}} = \sqrt{C(r)},$$

$$R_p = R_p(r) = \sqrt{(|r-r_0|^n+\alpha^n)^{\frac{1}{n}}\left((|r-r_0|^n+\alpha^n)^{\frac{1}{n}}-\alpha\right)} + \alpha \ln\left|\frac{\sqrt{(|r-r_0|^n+\alpha^n)^{\frac{1}{n}}}+\sqrt{(|r-r_0|^n+\alpha^n)^{\frac{1}{n}}-\alpha}}{\sqrt{\alpha}}\right|,$$

$$n \in \Re^+, \quad r \in \Re, \quad r \neq r_0, \tag{7}$$

where n and r_0 are entirely arbitrary constants. Equations (6), and their equivalents, equations (7), are most important, as they contain the solution to the confusions of the black hole and big bang relativists about radii, horizons, and singularities. The point-mass is always located in the gravitational field where the proper radius is zero, at $R_p(r_0) = 0\ \forall\ r_0$, and this has an associated radius of curvature of $R_c(r_0) = 2m\ \forall\ r_0$, in the case of the static, vacuum Schwarzschild field. $R_c(r_0) = 2m$ *does not denote a horizon or a trapped surface.* It is the minimum possible radius of curvature and it is a scalar invariant. Let's choose $n = 1,\ r_0 = \alpha,\ r > r_0$. Then

$$R_c(D) = D + \alpha = |r - r_0| + \alpha = (r-\alpha) + \alpha = r = R_c(r), \tag{8}$$

which yields the so-called "Schwarzschild" solution, thus

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1-\frac{\alpha}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{9}$$

But clearly, according to (5) and (8), $D \to 0$ when $r \to \alpha = r_0$ for *this particular choice* of n and r_0. Recall that r_0 denotes the parametric location of the point-mass in Minkowski space and $R_p(r_0) = 0$ the invariant location of the point-mass in the gravitational field. It is the choice of $n = 1,\ r_0 = \alpha,\ r > \alpha$ that makes α drop out of the radius of curvature in expression (9), as shown by expression (8), but that does not mean that the point-mass is not located at $r_0 = \alpha$ in parameter space in this particular case. Hence, on (9), $\alpha < r < \infty$, and there is no interior and no horizon. To emphasize this let's rewrite (9) using (8), thus

$$ds^2 = \left(1 - \frac{\alpha}{[(r-\alpha)+\alpha]}\right)dt^2 - \left(1 - \frac{\alpha}{[(r-\alpha)+\alpha]}\right)^{-1}\{d[(r-\alpha)+\alpha]\}^2 - [(r-\alpha)+\alpha]^2(d\theta^2+\sin^2\theta d\varphi^2)$$

$$= \left(1 - \frac{\alpha}{(D+\alpha)}\right)dt^2 - \left(1 - \frac{\alpha}{(D+\alpha)}\right)^{-1}[d(D+\alpha)]^2 - (D+\alpha)^2(d\theta^2+\sin^2\theta d\varphi^2).$$

Then, $r \to r_0 = \alpha \Rightarrow D \to 0$, and $R_p \to 0$ and $R_c \to \alpha = 2m$. Hence, there is no interior and no horizon associated with (9), because when $r = r_0 = \alpha$ on (9), the parametric distance D between point-mass and test particle goes to zero in Minkowski space, and so the proper distance (i.e. the proper radius) goes to zero and the radius of curvature goes to $\alpha = 2m$ in the gravitational field. This results for *any* choice of n and r_0, and thus for all particular radii of curvature obtained from equations (6) or (7).

Now let us just *assume*, as the black holers and big bangers do, (for the sake of argument), that r can go down to $r = 0$ on (9) to produce a "physical" singularity there. Then by the general expressions (7),

$$D = |0 - \alpha| = \alpha,$$

and so $R_c(D = \alpha) = \alpha + \alpha = 2\alpha$ and $R_p(D = \alpha) > 0$. In other words, the test particle has passed right through the point-mass located at the parametric *point* $r_0 = \alpha$ and out to a distance $D = \alpha$ from the point-mass, in parameter space, on the other side of the point-mass, and hence $R_p(D = \alpha) > 0$ in the gravitational field. (This scenario is of course impossible, since the line element is undefined at the location of the point-mass, because it looses Lorentz signature there.) Again, there is no interior and no horizon either in parametric Minkowski space or in the gravitational field. Horizons are utter rubbish and so the point-mass is not located at a "horizon" either. Consequently, the Kruskal-Szekeres extension is mathematical gibberish - completely meaningless. It is an ad hoc construction to get inside a non-existent interior bounded by a non-existent horizon or trapped surface. The Kruskal-Szekeres "extension" is a completely different pseudo-Riemann manifold which has nothing whatsoever to do with the gravitational field. The whole concept is a product of gross incompetence in elementary differential geometry, pure and simple. The consequences are fatal to the claims for black holes, the expanding Universe, and the big bang. The Friedmann models, the de Sitter model, the FRW models, Einstein's cylindrical model, are all invalid. Furthermore, it easily follows that cosmological solutions for Einstein's gravitational field on spherically symmetric, isotropic, type 1 Einstein spaces do not even exist!

To emphasize that only the relationship between the parametric distance $D = |r - r_0|$ and the gravitational radii $R_p(D)$ and $R_c(D)$ is important, where $R_p(D = 0) = 0$ and $R_c(D = 0) = \alpha$ are scalar invariants for the Schwarzschild space of the fictitious point-mass, I write the line element thus

$$ds^2 = \left(1 - \frac{\alpha}{R_c(D)}\right) dt^2 - \left(1 - \frac{\alpha}{R_c(D)}\right)^{-1} dR_c(D)^2 - R_c(D)^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$R_c(D) = (D^n + \alpha^n)^{\frac{1}{n}}\ ,\ \alpha = 2m,\ D = |r - r_0|,$$

$$n \in \Re^+,\ r \in \Re,\ r \neq r_0, \tag{10a}$$

where n and r_0 are entirely arbitrary. The proper radius for this metric is

$$R_p(D) = \sqrt{R_c(D)\left(R_c(D) - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c(D)} + \sqrt{R_c(D) - \alpha}}{\sqrt{\alpha}}\right)$$

$$= \sqrt{(D^n + \alpha^n)^{\frac{1}{n}}\left[(D^n + \alpha^n)^{\frac{1}{n}} - \alpha\right]} + \alpha \ln\left(\frac{\sqrt{(D^n + \alpha^n)^{\frac{1}{n}}} + \sqrt{(D^n + \alpha^n)^{\frac{1}{n}} - \alpha}}{\sqrt{\alpha}}\right). \tag{10b}$$

Then when $D = 0$, $R_p(D = 0) = 0$ and $R_c(D = 0) = \alpha\ \forall\ r_0$ and $\forall\ n$, since $D = |r - r_0|$. Now let $r_0 = \alpha = 2m$, so $D = |r - 2m|$, as in the case of the black holers. Then $D = 0$ when $r = 2m$, but $D = 2m$ when $r = 0$. Thus, when the black holers' test particle passes through their alleged "horizon" (where $D = 0$ and $R_p = 0$!), D *is no longer zero*, which means that the test particle is at a distance $D > 0$ from the point-mass on the other side of the point-mass in Minkowski parameter space, and similarly in the gravitational field since $D > 0 \Rightarrow R_p(D) > 0$. The test particle can approach or recede from the point-mass from above r_0 or below r_0 in parameter space, but not from above and below r_0 simultaneously. Hence, there are no interiors and no horizons, and no black holes. Equations (10a) and (10b) completely eliminate the black hole since the alleged black hole singularity is completely removed because these equations are well-defined on $-\infty < r_0 < \infty$ irrespective of the value assigned to r_0, and the sole singularity arising at r_0 invariantly produces $g_{00}(r_0) = 0$. There is no possibility whatsoever for $g_{11} = 0$. Einstein and Rosen tried to find a solution with a somewhat similar property, in relation to the so-called "Schwarzschild" solution, manifest as the Einstein-Rosen bridge, but they horribly botched it, as pointed out in my papers. Their failure was due to the fact that they did not understand the intrinsic geometry of the line-element, just like the rest of the black hole and big bang relativists. Note also that the "wormhole" is also obliterated.

It is easily proved that there are no curvature-type singularities in Einstein's gravitational field. Consider the general solution of equation (1). The Kretschmann scalar is given by $f = R_{\alpha\beta\rho\sigma}R^{\alpha\beta\rho\sigma}$, and for equation (1) this gives

$$f = \frac{12\alpha^2}{C^3(r)}.$$

Then by (7),

$$f = \frac{12\alpha^2}{\left(|r - r_0|^n + \alpha^n\right)^{\frac{6}{n}}}.$$

Consequently,

$$f(r_0) \equiv \frac{12}{\alpha^4},$$

irrespective of the actual value of r_0. Also note that

$$\lim_{r \to \infty^{\pm}} f(r) = 0.$$

Thus, the Kretschmann scalar is finite everywhere.

The singularity at $R_p(r_0) = 0^+$ is insurmountable because,

$$\lim_{r \to r_0^{\pm}} \frac{2\pi R_c(r)}{R_p(r)} = \infty.$$

All the details of the mathematical arguments that invalidate the black hole in all its flavours can be had at

On Expansion of the Universe and Big Bang

In Section 2 of ECPF, Kiefer and Polarski simply postulate the existence of a Friedmann Universe. However, if follows from the forgeoing discussion on black holes that the true geometry of Einstein's gravitational field does not support Friedmann's models and the Standard Cosmological Model. Indeed, it is easily proved that cosmological solutions for Einstein's gravitational field for isotropic, spherically symmetric, type 1 Einstein spaces, do not exist! Therefore, there is currently no valid General Relativistic cosmological model at all. The Friedmann models are invalid, and so there is no big bang. Furthermore, it is easily proved that de Sitter's spherical Universe and Einstein's cylindrical Universe are also fallacious. Consequently the entire paper by Kiefer and Polarski is erroneous, and with it the work of Hehl and his group on relativistic cosmology, as I now prove.

The line-element obtained by the Abbé Lemaitre and by Robertson, for instance, is inadmissible. Consider the metric

$$ds^2 = \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} r^2\right) dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} r^2\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \tag{11}$$

This can be written in general as,

$$ds^2 = \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} C\right) dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3} C\right)^{-1} \frac{C'^2}{4C} dr^2 - \ C \left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \tag{12}$$

$$C = C(D(r)), \quad D(r) = |r - r_0|, \quad r_0 \in \Re ,$$

where r_0 is arbitrary.

Under the false assumption that r is a radius in de Sitter's spherical universe, they proposed the following transformation of coordinates on the metric (11),

$$\bar{r} = \frac{r}{\sqrt{1 - \frac{r^2}{W^2}}} e^{-\frac{t}{W}}, \quad \bar{t} = t + \frac{1}{2} W \ln\left(1 - \frac{r^2}{W^2}\right) , \tag{13}$$

$$W^2 = \frac{\lambda + 8\pi\rho_{00}}{3} ,$$

to get

$$ds^2 = d\bar{t}^2 - e^{\frac{2\bar{t}}{W}} \left(d\bar{r}^2 + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta d\varphi^2\right) ,$$

or, by dropping the bar and setting $k = \frac{1}{W}$,

$$ds^2 = dt^2 - e^{2kt} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right) . \tag{14}$$

Now the most general non-static line-element can be written as,

$$ds^2 = A(D,t) dt^2 - B(D,t) dr^2 - C(D,t) \left(d\theta^2 + \sin^2\theta d\varphi^2\right) ,$$

$$D = |r - r_0|, \quad r_0 \in \Re$$

or, since $dD^2 = dr^2$,

$$ds^2 = A(D,t) dt^2 - B(D,t) dD^2 - C(D,t) \left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \tag{15}$$

$$D = |r - r_0|, \quad r_0 \in \Re$$

where analytic $A, B, C > 0 \ \forall \ r \neq r_0$ and $\forall \ t$.

Rewrite (15) by setting,

$$A(D,t) = e^{\nu}, \quad \nu = \nu(G(D), t) ,$$

$$B(D,t) = e^{\sigma}, \quad \sigma = \sigma(G(D), t) ,$$

$$C(D,t) = e^{\mu} G^2(D), \quad \mu = \mu(G(D), t) ,$$

to get

$$ds^2 = e^{\nu} dt^2 - e^{\sigma} dG^2 - e^{\mu} G^2(D) \left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \tag{16}$$

Now set,

$$r^* = G(D(r)) , \tag{17}$$

to get

$$ds^2 = e^{\nu} dt^2 - e^{\sigma} dr^{*2} - e^{\mu} r^{*2} \left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \tag{18}$$

$$\nu = \nu(r^*, t) , \quad \sigma = \sigma(r^*, t) , \quad \mu = \mu(r^*, t) .$$

One then finds in the usual way that the solution to (18) is,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4} r^{*2}\right)^2} \left[dr^{*2} + r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \tag{19}$$

where k is a constant.

Then by (17) this becomes,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4} G^2\right)^2} \left[dG^2 + G^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] ,$$

or,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4} G^2\right)^2} \left[G'^2 dr^2 + G^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \tag{20}$$

$$G' = \frac{dG}{dr} ,$$

$$G = G(D(r)) , \quad D(r) = |r - r_0| , \quad r_0 \in \Re .$$

The admissible form of $G(D(r))$ must now be determined.

If $G' \equiv 0$, then $B(D,t) = 0 \ \forall\ r$ and $\forall\ t$, in violation of (15). Therefore $G' \neq 0 \ \forall\ r \neq r_0$.

Metric (20) is singular when,

$$1 + \frac{k}{4} G^2(r_0) = 0 ,$$

$$\Rightarrow \ G(r_0) = \frac{2}{\sqrt{-k}} \Rightarrow k < 0 . \tag{21}$$

The proper radius on (20) is,

$$R_p(r,t) = e^{\frac{1}{2}g(t)} \int \frac{dG}{1 + \frac{k}{4} G^2} = e^{\frac{1}{2}g(t)} \left(\frac{2}{\sqrt{k}} \arctan \frac{\sqrt{k}}{2} G(r) + K\right) ,$$

$$K = \text{const} ,$$

which must satisfy the condition,

$$\text{as } r \to r_0^{\pm} , \ R_p \to 0^+ .$$

Therefore,

$$R_p(r_0,t) = e^{\frac{1}{2}g(t)} \left(\frac{2}{\sqrt{k}} \arctan \frac{\sqrt{k}}{2} G(r_0) + K\right) = 0 ,$$

and so

$$R_p(r,t) = e^{\frac{1}{2}g(t)} \frac{2}{\sqrt{k}} \left[\arctan \frac{\sqrt{k}}{2} G(r) - \arctan \frac{\sqrt{k}}{2} G(r_0)\right] . \tag{22}$$

Then by (21),

$$R_p(r,t)=e^{\frac{1}{2}g(t)}\frac{2}{\sqrt{k}}\left[\arctan\frac{\sqrt{k}}{2}G(r)-\arctan\sqrt{-1}\right], \tag{23}$$

$$k<0\,.$$

Therefore, there exists no function $G(D(r))$ rendering a solution to (20) in the required form of (15).

The relativists however, owing to their invalid assumptions about the parameter r, write equation (20) as,

$$ds^2=dt^2-\frac{e^{g(t)}}{\left(1+\frac{k}{4}r^2\right)^2}\left[dr^2+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right], \tag{24}$$

having assumed that $G(D(r))\equiv r$, and erroneously take r as a radius on the metric (24), valid down to $r=0$. Metric (24) is a meaningless concoction of mathematical symbols. Nevertheless, the relativists transform this meaningless expression with a meaningless change of "coordinates" to obtain the Robertson-Walker line-element, as follows.

Transform (20) by setting,

$$\bar{G}(\bar{r})=\frac{G(r)}{1+\frac{k}{4}G^2}\,.$$

This carries (20) into,

$$ds^2=dt^2-e^{g(t)}\left[\frac{d\bar{G}^2}{\left(1-\kappa\bar{G}^2\right)}+\bar{G}^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right]. \tag{25}$$

This is easily seen to be the familiar Robertson-Walker line-element if, following the relativists, one incorrectly assumes $\bar{G}\equiv\bar{r}$, disregarding the fact that the admissible form of $\bar{G}$ *must be ascertained*. In any event (25) (and therefore the usual FRW metric) is meaningless, owing to the meaninglessness of (20), which I confirm as follows.

$$\bar{G}'\equiv 0\Rightarrow\bar{B}=0\quad\forall\quad\bar{r}\,,$$

in violation of (15). Therefore

$$\bar{G}'\neq 0\quad\forall\quad\bar{r}\neq\bar{r}_0.$$

Equation (25) is singular when,

$$1-k\bar{G}^2(\bar{r}_0)=0\;\Rightarrow\bar{G}(\bar{r}_0)=\frac{1}{\sqrt{k}}\;\Rightarrow k>0\,. \tag{26}$$

The proper radius on (25) is,

$$\bar{R}_p=e^{\frac{1}{2}g(t)}\int\frac{d\bar{G}}{\sqrt{1-k\bar{G}^2}}=e^{\frac{1}{2}g(t)}\left(\frac{1}{\sqrt{k}}\arcsin\sqrt{k}\bar{G}(\bar{r})+K\right),$$

$$K=\text{const.}\,,$$

which must satisfy the condition,

$$\text{as}\quad\bar{r}\to\bar{r}_0^{\pm},\;\bar{R}_p\to 0^+\,,$$

so

$$\bar{R}_p(\bar{r}_0,t)=0=e^{\frac{1}{2}g(t)}\left(\frac{1}{\sqrt{k}}\arcsin\sqrt{k}\bar{G}(\bar{r}_0)+K\right).$$

Therefore,

$$\bar{R}_p(\bar{r},t)=e^{\frac{1}{2}g(t)}\frac{1}{\sqrt{k}}\left[\arcsin\sqrt{k}\bar{G}(\bar{r})-\arcsin\sqrt{k}\bar{G}(\bar{r}_0)\right]. \tag{27}$$

Then

$$\sqrt{k}\bar{G}(\bar{r}_0)\leqslant\sqrt{k}\bar{G}(\bar{r})\leqslant 1\,,$$

or

$$\bar{G}(\bar{r}_0) \leqslant \bar{G}(\bar{r}) \leqslant \frac{1}{\sqrt{k}} \,.$$

Then by (26),

$$\frac{1}{\sqrt{k}} \leqslant \bar{G}(\bar{r}) \leqslant \frac{1}{\sqrt{k}} \,,$$

so

$$\bar{G}(\bar{r}) \equiv \frac{1}{\sqrt{k}} \,.$$

Consequently, $\bar{G}'(\bar{r}) = 0 \ \forall \ \bar{r}$ and $\forall \ t$, in violation of (15). Therefore, there exists no function $\bar{G}(\bar{D}(\bar{r}))$ to render a solution to (25) in the required form of (15).

If the conditions on (15) are relaxed in the fashion of the relativists, non-Einstein metrics with expanding radii of curvature are obtained. Nonetheless the associated spaces have zero volume. Indeed, equation (14) becomes,

$$ds^2 = dt^2 - e^{2kt}\,\frac{\left(\lambda + 8\pi\rho_{00}\right)}{3}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \,. \tag{28}$$

This is not an Einstein universe. The radius of curvature of (28) is,

$$R_c(r,t) = e^{kt}\sqrt{\frac{\lambda + 8\pi\rho_{00}}{3}} \,,$$

which expands or contracts with the sign of the constant k. Even so, the proper radius of the "space" of (28) is,

$$R_p(r,t) = \lim_{r \to \pm\infty} \int_{r_0}^{r} 0 \ dr \equiv 0 \,.$$

The volume of this point-space is,

$$V = \lim_{r \to \pm\infty} e^{2kt}\,\frac{\left(\lambda + 8\pi\rho_{00}\right)}{3} \int_{r_0}^{r} 0 \ dr \int_{0}^{\pi} \sin\theta \ d\theta \int_{0}^{2\pi} d\varphi \equiv 0 \,.$$

Metric (28) consists of a single "world line" through the point $R_p(r,t) \equiv 0$. Furthermore, $R_p(r,t) \equiv 0$ is an insurmountable singular "point-space" since the ratio,

$$\frac{2\pi e^{kt}\sqrt{\lambda + 8\pi\rho_{00}}}{\sqrt{3}R_p(r,t)} \equiv \infty \,.$$

Therefore, $R_p(r,t) \equiv 0$ cannot be extended.

Similarly, equation (25) becomes,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{k}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \,, \tag{29}$$

which is not an Einstein metric. The radius of curvature of (29) is,

$$R_c(r,t) = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} \,,$$

which changes with time. The proper radius is,

$$R_p(r,t) = \lim_{r \to \pm\infty} \int_{r_0}^{r} 0 \ dr \equiv 0 \,,$$

and the volume of the point-space is

$$V = \lim_{r \to \pm\infty} \frac{e^{g(t)}}{k} \int_{r_0}^{r} 0 \; dr \int_{0}^{\pi} \sin\theta \; d\theta \int_{0}^{2\pi} d\varphi \equiv 0 \, .$$

Metric (24) consists of a single "world line" through the point $R_p(r,t) \equiv 0$. Furthermore, $R_p(r,t) \equiv 0$ is an insurmountable singularity since the ratio,

$$\frac{2\pi e^{\frac{1}{2}g(t)}}{\sqrt{k} R_p(r,t)} \equiv \infty \, .$$

Therefore, $R_p(r,t) \equiv 0$ cannot be extended.

It immediately follows that the Friedmann models are all invalid, because the so-called Friedmann equation, with its associated equation of continuity, $T^{\mu\nu}_{\;;\mu} = 0$, is based upon metric (20), which, as I have proven, has *no solution* in $\overline{G}(\overline{r})$ in the required form of (15). Furthermore, metric (24) cannot represent an Einstein universe and therefore has no cosmological meaning. Consequently, the Friedmann equation is also nothing more than a meaningless concoction of mathematical symbols, destitute of any physical significance whatsoever. Friedmann incorrectly assumed, just as the relativists have done all along, that the parameter r is a radius in the gravitational field. Owing to this erroneous assumption, his treatment of the metric for the gravitational field violates the inherent geometry of the metric and therefore violates the geometrical form of the pseudo-Riemannian spacetime manifold. The same can be said of Einstein himself, who did not understand the geometry of his own creation, and by making the same mistakes, failed to understand the implications of his theory.

Thus, the Friedmann models are all invalid, as is the Einstein-de Sitter model, and all other general relativistic cosmological models purporting an expansion of the universe. Furthermore, there is no general relativistic substantiation of the Big Bang hypothesis. The static cosmological isotropic models are invalid for the very same reasons, as is easily confirmed. Since the Big Bang hypothesis rests solely upon an invalid interpretation of General Relativity, it is invalid. The standard interpretations of the Hubble-Humason relation and the cosmic microwave background are not consistent with Einstein's theory either.

The full mathematical details can be obtained at

www.geocities.com/theometria/papers.html

Conclusion

Professor Hehl and his colleagues have fatally erred in their anaylsis of black holes and relativistic cosmology. They are of course not alone, since they toe the standard line of spurious argument.

Stephen J. Crothers

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