On the ECE Lemma

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For the first time the 'ECE Lemma' appeared in 2003 in the FOPL paper [1] THE EVANS LEMMA OF DIFFERENTIAL GEOMETRY . There the essential lines are

. . . where the scalar curvature R_1 is defined by

$$-R_1 q^a{}_\lambda := (D^\mu \omega_\mu{}^a{}_b)\, q^b{}_\lambda - (D^\mu \Gamma^\nu{}_{\mu\lambda})\, q^a{}_\lambda \,. \qquad (44)$$

. . .

Define the scalar curvature R_2 by

$$-R_2 q^a{}_\lambda := -\Gamma^\nu{}_{\nu\mu} \omega^{\mu a}{}_b\, q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu}\, \Gamma^\mu{}_\lambda{}^\nu\, q^a{}_\nu \,, \qquad (48)$$

to obtain the Evans lemma

$$\Box q^a{}_\lambda = R\, q^a{}_\lambda, \qquad (49)$$

where

$$R = R_1 + R_2. \qquad (50)$$

The *problem* with these definitions is that there are *two indices* on the left hand side of the eqs. (44) and (48) running each over 0,1,2,3. Therefore each equation (44) and (48) de facto represents 4×4 = 16 definitions of R_1 and R_2 respectively. Since the author does not worry about the compatibility of his 2×16 definitions we must conclude that in reality the quantities R_1 and R_2 (and so R, required for the ECE Lemma (49)) are *not at all well-defined* by the eqs. (44) and (48).

Thus there is no valid proof of of the ECE Lemma (49) in the paper [1].

Later on the author tried to give other proofs for the ECE Lemma:

In the web note [2] we find a simplification of the consideration [1], now supplied with an attempt to close the gap mentioned above:

. . . i.e.

$$\Box\, q^a{}_\lambda = \partial^\mu (\Gamma^\nu{}_{\mu\lambda}\, q^a{}_\nu - \omega^a{}_{\mu b}\, q^b{}_\lambda) \qquad (8)$$

Now define

$$R = q_a{}^\lambda\, \partial^\mu (\Gamma^\nu{}_{\mu\lambda}\, q^a{}_\nu - \omega^a{}_{\mu b}\, q^b{}_\lambda) \qquad (9)$$

and use

$$q^a{}_\lambda q^\lambda{}_a = 1 \quad \text{(error! = 4 would be correct.)} \qquad (10)$$

to find by using the eqs. (8-10)

$$\begin{aligned} \Box\, q^a{}_\lambda &=_{(8)} 1 \cdot \partial^\mu (\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda) \\ &=_{(10)} [q^a{}_\lambda q^\lambda{}_a] \cdot \partial^\mu (\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda) \\ &=_{(?)} q^a{}_\lambda \cdot [q^\lambda{}_a\, \partial^\mu (\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda)] \\ &=_{(9)} R\, q^a{}_\lambda\,. \end{aligned} \qquad (11)$$

The choice of the indices $^a{}_\lambda$ and $^\lambda{}_a$ in $[q^a{}_\lambda q^\lambda{}_a]$ as dummy indices is ***inadmissible*** due to the occurrence of a and λ in the next expression (...). If other correct dummies are used instead then the next step $=_{(?)}$ is ***impossible*** . Hence the calculation (11) is *invalid.* However the calculation above is quite following the author's ideas of New Math.

Summary: There is no valid proof of the ECE Lemma.

References

[1] M.W. Evans, *THE EVANS LEMMA OF DIFFERENTIAL GEOMETRY*, http://www.aias.us/documents/uft/a7thpaper.pdf

[2] M.W. Evans, *Some Key Derivations: 1. Derivations of the Lemma* , http://www.atomicprecision.com/blog/wp-filez/akeyderivations1and2.pdf

[3] G.W. Bruhn, *The ECE Lemma, Comments*, http://www.mathematik.tu-darmstadt.de/~bruhn/ECE-Lemma011007.html

[4] G.W. Bruhn, *A remark on Evans' recent web article on the ECE Lemma*, http://www.mathematik.tu-darmstadt.de/~bruhn/toMWE240507.html