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On the Non-Lorentz-Invariance of M.W. EVANS' $O(3)$-Symmetry Law

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Abstract In 1992 M.W. EVANS proposed the $O(3)$ symmetry of electromagnetic fields by adding a constant longitudinal magnetic field to the well-known transverse electric and magnetic fields of circularly polarized plane waves, such that certain cyclic relations of a so-called $O(3)$ symmetry are fulfilled. Since then M.W. EVANS has elevated this $O(3)$ symmetry to the status of a *new law of electromagnetics*. As a law of physics must be invariant under admissible coordinate transforms, namely Lorentz transforms, in 2000 he published a proof of the Lorentz invariance of $O(3)$ symmetry of electromagnetic fields. As will be shown below this proof is incorrect; more, after simple correction it will turn out here that the $O(3)$ symmetry cannot be Lorentz invariant.

1 M.W. EVANS' $O(3)$ Hypothesis

The assertion of $O(3)$ symmetry of electromagnetic fields is a central concern of M.W. EVANS' research since 1992: He claims that the transverse magnetic field of a circularly polarized electromagnetic plane wave is accompanied by a constant longitudinal field $\mathbf{B}^{(3)}$, the so-called "ghost field":

M.W. EVANS considers a circularly polarized plane electromagnetic wave propagating along the z axis of a Cartesian coordinate system [1, Chap. 1.2]. Using the electromagnetic phase

[1, (38)][1] $$\Phi = \omega t - \kappa z,$$

[1]Equations from M.W. EVANS' book on the so-called Grand Covariant Unified Field Theory (GCUFT) [1] appear with equation labels [1, (nn)] in the left margin.

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where t denotes time, $\kappa = \omega/c$ is the free-space wave number, ω is the angular frequency, and c is the speed of light in free space, M.W. EVANS describes the wave in terms of his complex circular basis [1, (1.41)]. The total magnetic field is stated by him as

[1, (43/1)] $$\mathbf{B}^{(1)} = \frac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} - i\mathbf{j}) e^{i\Phi},$$

[1, (43/2)] $$\mathbf{B}^{(2)} = \frac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} + i\mathbf{j}) e^{-i\Phi},$$

[1, (43/3)] $$\mathbf{B}^{(3)} = B^{(0)} \mathbf{k},$$

where $i = \sqrt{-1}$ and **i**, **j**, and **k** are the Cartesian unit vectors. The total magnetic field satisfies his "cyclic $O(3)$ symmetry relations"

[1, (44/1)] $$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = i B^{(0)} \mathbf{B}^{(3)*},$$

[1, (44/2)] $$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = i B^{(0)} \mathbf{B}^{(1)*},$$

[1, (44/3)] $$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = i B^{(0)} \mathbf{B}^{(2)*}.$$

Equation [1, (43/3)] specifically *defines* EVANS' ghost field $\mathbf{B}^{(3)}$, which is coupled by the relations [1, (44)] to the transverse components stated as [1, (43/1)] and [1, (43/2)].

M.W. EVANS' B Cyclic Theorem is the statement that the magnetic field components [1, (43/1)] and [1, (43/2)] of each circularly polarized plane wave are accompanied by a longitudinal component [1, (43/3)], and that all three components together fulfill the cyclic $O(3)$ symmetry relations [1, (44)]. EVANS considers this $O(3)$ hypothesis as a **Law of Physics**.

2 Is the $O(3)$ Hypothesis Lorentz-Invariant?

A law of physics must be invariant under admissible coordinate transforms, namely Lorentz transforms. A circularly polarized plane wave displays this property when described by any observer who is moving with uniform velocity with respect to an inertial frame K at rest. Therefore, EVANS' $O(3)$ symmetry law should be valid in all inertial frames of reference. Hence, to check the physical validity of EVANS' $O(3)$ hypothesis, we shall apply a longitudinal Lorentz transform to the plane wave as described by EVANS (the ghost field included).

In an article published in 2000 [2, p. 14], EVANS provided a proof of the Lorentz invariance of the $O(3)$ hypothesis [1, (44)] by referring to the invariance of the vector potential $\mathbf{A}$ under Lorentz transforms. That is a good method obtaining the transformed magnetic field, *if done correctly*.[2]

The vector potentials of the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ of the plane wave under consideration are given by

$$\left.\begin{aligned} \mathbf{A}^{(1)} &= \frac{1}{\kappa}\mathbf{B}^{(1)} = \frac{1}{\kappa\sqrt{2}}B^{(0)}(\mathbf{i} - i\mathbf{j})e^{i\Phi} \\ \mathbf{A}^{(2)} &= \frac{1}{\kappa}\mathbf{B}^{(2)} = \frac{1}{\kappa\sqrt{2}}B^{(0)}(\mathbf{i} + i\mathbf{j})e^{-i\Phi} \end{aligned}\right\}, \tag{1}$$

while the vector potential of the *longitudinal* component $\mathbf{B}^{(3)}$ is

$$\mathbf{A}^{(3)} = \frac{1}{2}\mathbf{B}^{(3)} \times (x\mathbf{i} + y\mathbf{j}) = \frac{1}{2}B^{(0)}(x\mathbf{j} - y\mathbf{i}). \tag{2}$$

The Lorentz invariance of the vector potential $\mathbf{A}^{(3)}$ yields the Lorentz invariance of $\mathbf{B}^{(3)}$ and the factor $B^{(0)}$ in [1, (43)] and [1, (44)] for *longitudinal* Lorentz transforms (i.e. between inertial frames K and K' such that K' moves relative to K with velocity $\mathbf{v} \parallel \mathbf{k}$ and $\beta = |\mathbf{v}|/c$).

EVANS [2] ignored that ω and κ are *not* Lorentz-invariant. Under *longitudinal* Lorentz transforms, we have the well-known Doppler effect:

$$\omega' = \sqrt{\frac{1-\beta}{1+\beta}}\omega, \qquad \kappa' = \sqrt{\frac{1-\beta}{1+\beta}}\kappa. \tag{3}$$

Therefore the invariance of the vector potentials *does not transfer* to the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$.

More importantly here, due to the invariance of $\mathbf{A}$ we obtain

$$\mathbf{B}'^{(1)} \times \mathbf{B}'^{(2)} = \kappa'^2\mathbf{A}'^{(1)} \times \mathbf{A}'^{(2)} = \frac{1-\beta}{1+\beta}\,\kappa^2\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{1-\beta}{1+\beta}\,\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}, \tag{4}$$

from (1). That is, the expression $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ on the left side of [1, (44/1)] is not Lorentz-invariant, whereas $\mathbf{B}^{(3)}$ on the right side of [1, (44/1)] is Lorentz-invariant due to (2). Equation [1, (44/1)] therefore, if valid in the inertial frame K, *cannot be valid* also in the inertial frame K'. Hence, EVANS' cyclic $O(3)$ symmetry relations [1, (44)] are *not Lorentz-invariant and so cannot be a Law of Physics*.

3 Epilogue

After the foregoing sections had appeared on http://arxiv.org, EVANS replied on www.atomicprecision.com/blog/2006/12/26/jackson-on-the-lorentz-transformation/.

[2] That EVANS' $O(3)$ symmetry is not Lorentz-invariant was a conclusion already derived in [4]. In that paper I gave an independent proof of the non-Lorentz invariance of the $O(3)$ symmetry based on the well-known Lorentz transformation of the electromagnetic fields, while here flawed methods proposed by M.W. EVANS himself are corrected to obtain the same result.

Therein, he suggested that Equation (3.111) of another book [3] of his should prove the Lorentz-invariance of his "B Cyclics".

This equation is as follows:

[3, (3.111)] $$B^{(0)\prime} = \left[\frac{1 - v/c}{1 + v/c}\right]^{1/2} B^{(0)}.$$

It is a consequence of the Lorentz transform applied to the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$. However, just before [3, (3.111)]) Evans stated the transformation rule

[3, (3.110-3)] $$\mathbf{B}^{(3)\prime} = \mathbf{B}^{(3)}, \ldots$$

This rule implies $|\mathbf{B}^{(3)\prime}| = |\mathbf{B}^{(3)}|$ which immediately leads to a *contradiction* in Ref. [3] itself, as on p. 5 of that book we find the equation

[3, (1.2a)] $$|\mathbf{B}^{(1)}| = |\mathbf{B}^{(2)}| = |\mathbf{B}^{(3)}| = B^{(0)}$$

that yields

$$B^{(0)\prime} = |\mathbf{B}^{(3)\prime}| = |\mathbf{B}^{(3)}| = B^{(0)}$$

in contrast to [3, (3.111)]. Therefore, EVANS stopped his consideration in Ref. [3] just before recognizing the final contradiction he would have arrived at.

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