In press at a collection of papers on dark matter

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This paper is dedicated to my newly born granddaughter Tatiana Santilli,

hoping that one day she can understand its content

APPARENT ABSENCE OF DARK MATTER DUE TO DEVIATIONS FROM THE SPEED OF LIGHT IN THE INTERIOR OF STARS, QUASARS AND BLACK HOLES

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Abstract

We recall the validity of *special relativity* for the physical conditions it was intended for, dynamics in in vacuum, and show its inapplicability (rather than violation) for physical conditions it was not intended for, such as for the classical treatment of antimatter, interior dynamical problems within physical media, and matter in irreversible conditions. We outline the covering relativity constructed and verified during the past four decades by numerous scholars, today known as *isorelativity,* that resolves the above limitations via an invariant representation of arbitrary speeds $C = c/n$ in the universe. We submit the hypothesis that *dark matter* and *dark energy* are due to the use of the law $E = mc^2$ for vacuum, while their existence is removed by the use of the isorelativistic law $E = mC^2 = mc^2/n^2$ because the average value of C for the interior of hyperdense astrophysical bodies, such as stars, quasars and black holes, results to be a large multiple of c. The study is complemented with the introduction of *dark antimatter* and the study of both dark matter and antimatter in irreversible conditions.

1. Inapplicability of special relativity to dark matter

Special relativity has a majestic axiomatic structure and an impressive experimental verification for the physical conditions clearly identified by its Founders [1a-1e], for electromagnetic waves and point-particles moving in vacuum (hereon simply intended as empty space). Outside these conditions of original conception and verification, special relativity is generally *inapplicable,* and not "violated" because not conceived for broader conditions. Contrary to popular beliefs, there exist numerous physical conditions under which special relativity is inapplicable. Those with a progressive direct relevance for *dark matter* (see [1i] for a general review and primary literature) are the following:

ANTIMATTER [6n]. It has remained virtually unknown in the technical literature of the 20-th century that *both special and general relativity are unable to provide any meaningful classical representation of antimatter.* In fact, said relativities (see [2] for historical contributions on gravitation) can only represent antimatter via the change of the sign of the charge (rather than the correct anti-isomorphic map discussed in the next section). As a result, they admit no distinction whatever between neutral matter and antimatter. Even for charged antiparticle we have structural inconsistencies because the operator image of such a classical representations is given by a "particle" with the wrong sign of the charge and cannot possibly be the needed "antiparticle" due to the uniqueness of the quantization channel.

Rather then being innocuous, this limitation of both special and general relativity has serious astrophysical implications. To begin, antimatter exists in the universe to such an extent that half of the universe could be made of up antimatter according to new cosmologies [5f,5h] that exhibit no discontinuity at the time of its creation. Irrespective of whether this is true or false, we must assume the existence of *dark antimatter* in the universe to avoid ascientific posturing. But then, "any" use of special relativity for dark antimatter has no physical sense evidently because it is neutral. Also, recent *classical* theories of antimatter indicated in the next section have established the *negative* character of the energy (when treated with the appropriate mathematics). This begins to illustrate the complexity of the very identification of the applicable notion of energy for dark matter and antimatter,

Moreover, the impossibility for a consistent representation of antimatter by special and general relativity has much deeper implications, such as being a main origin of the failure of achieving a grand unification until now since Einstein's times.

PHYSICAL MEDIA [6e-6i]. Another class of physical conditions directly relevant for both dark matter and antimatter in which special relativity is inapplicable can be generally defined as dealing with the so-called *interior dynamical problems,* or equivalently with the *dynamics within physical media,* such as planetary atmospheres, astrophysical chromospheres, or physical media at large. More importantly for the problem of dark matter and antimatter, the belief that special relativity is exactly valid within the hyperdense media in the interior of stars, galaxies and quasars, is ascientific due, not only to the lack of any verification of the relativity within the conditions considered, but also to the rather vast evidence establishing its inapplicability.

A direct way of seeing this inapplicability is by noting that the "universal constancy of the speed of light" c is a philosophical abstraction with quite damaging physical consequences illustrated in this note. On strict scientific grounds, the statement should solely and specifically refer to "the constancy of the speed of light in vacuum" to prevent numerous insufficiencies or sheer inconsistencies when referring to physical media such as:

1) The very existence of the speed of light within physical media, let alone its constancy, is outside scientific reason, *e.g.,* because physical media are generally opaque to light. The scientific issue is then the identification of a covering relativity with a new notion replacing the speed of light within physical media in general, and opaque media in particular, under the condition of recovering identically and uniquely special relativity and the speed of light c when considering the vacuum.

2) When physical media are transparent to light, its speed is generally different than c, thus establishing the lack of universality of the speed of light [6e,6f]. The general attitude in this case was that of decomposing light into photons, and interpreting said propagation as the result of the scattering of photons among the atoms of the medium, a reduction evidently done for the intent of recovering the validity of special relativity since photons would propagate in vacuum.

This belief has been disproved for both speeds smaller and bigger than c [6e,6f]. Typical cases for speeds smaller than c are given by the propagation of electromagnetic waves through our atmosphere. In this case there is no meaningful reduction to photons, *e.g.,* for radio waves with one meter wavelength.

Fully verified (but generally ignored) experimental evidence has established the propagation of electromagnetic waves at speeds bigger than c (see [10b] for comprehensive literature). For instance, even though the case continues to be ignored by the academic community, German experimentalists have transmitted an entire Beethoven symphony at speeds bigger than c via electromagnetic waves passing through certain guides. The reduction of light to photons scattering through atoms evidently loses any credibility in this case because photons propagate in vacuum at the speed c and their scattering among atoms could not possibly produce an overall speed bigger than c.

3) Yet another evidence establishing the lack of applicability of special relativity within physical media is visible to the naked eye, and it is given by the propagation of light in water. In this case light propagates at a speed of about $C = 2c_o/3$, while electrons can propagates in water at speeds bigger than the local speed of light (this event causes the well known Cerenkov light in the pools of nuclear reactors).

It is easy to see that the above setting causes the collapse of the basic principles of special relativity. In fact, if we assume C as the maximal causal speed in water, we have the loss of causality because ordinary electrons would travel faster than the local speed of light, the necessity of interpreting ordinary electrons as tachyons, and other inconsistencies [6e,6f].

If we assume that the speed of light *in vacuum* is the maximal causal speed *in water,* we have the collapse of other basic postulates of special relativity, such as the collapse of the relativistic law for composition of speeds [3]. In fact, in this case we have the relativistic sum of speeds

$$V_{tot} = (C + C)/(1 + C^2/c^2) = (2c/3 + 2c/3)/(1 + (2c/3)^2/c^2) = 12c/13. \quad (1.1)$$

namely, the collapse of a pillar of special relativity since the sum of two light speeds]it would not yield the light speed.

IRREVERSIBILITY [3j] . Another significant field of inapplicability of special relativity directly relevant to dark matter and antimatter is that for the events of our everyday life, since they are generally *irreversible in time,* (namely, their time reversal image is acausal), while all mathematical foundations of special relativity, let alone all its physical laws, are strictly *reversible in time* (namely, the image under time reversal is fully causal).

In reality, special relativity was correctly built by its Founders [1] to be reversible because that is a main feature of the events intended for treatment, such as the orbits of electrons in an atomic structure, with the clear understanding that the reversibility for such a class of systems positively cannot be extended to the entire universe. This inapplicability is established by the very existence of *thermodynamics* and its irreversible laws that are irreconcilably incompatible with special relativity, as well known by expert to qualify as such but not spoken.

Rather than being a mere curiosity, the inapplicability of special relativity for irreversible systems at large, and thermodynamics in particular, has a direct negative impact for serious studies on dark matter and antimatter to such an extend that, after solving the limitations caused by antimatter and those for interior problems, no conclusion is expected until the availability of a new covering relativity fully applicable to irreversible systems.

To further illustrate how serious the inapplicability is, it is known that special relativity is today one of the main obstacle toward the much needed search for new clean energies so much needed by our society in view of our deteriorating climate, trivially, because all energy releasing processes are strictly irreversible and the belief that they can be predicted and treated via a fully reversible relativity is ascientific. In fact, no truly new energy has been discovered under special relativity since the middle of the 20-th century while new studies beyond special relativity have already offered new predictions unthinkable with old theories.

As a result of all the above evidence, the only posible scientific conclusion is that inducated earlier, namely, *special relativity is inapplicable (rather than violated) for matter in general, let alone fdark matter and antimatter, thus establishing the need to search for a covering relativity.* In particular, the basic law $E = mc_o^2$ is not expected to be necessarilly applicable for the study of dark matter as used until now.

2. Santilli's isorelativity for physical media of matter and antimatter in reversible and irreversible conditions

To initiate our study of dark matter and the newly proposed antimatter, in this section we assume the conventional notion of mass and energy, and initially exclude dark antimatter and irreversible conditions, by restricting our initial attention to the most salient feature of dark matter, that of constituting an interior dynamical problem, or, equivalently, of being a physical medium. We shall later touch possibilities for adding antimatter and irreversible conditions.

The first relevant studies known to the author on the locally varying character of the speed of light are those by Lorentz [1e], whose studies are also quoted by Pauli [1f]. In essence, after achieving the invariance of the speed of light c in vacuum [1a], Lorentz attempted the identification of the invariance of the locally varying speed of light within physical media according to the familiar law

$$C = \frac{c}{n}, \tag{2.1}$$

where n is the known index of refraction. Unfortunately, Lorentz achieved historical results for the invariance of the speed of light c in vacuum [1a], but he failed to achieve a similar result for the locally varying speed $C = c/n$ [1e]for reasons today known, namely, the mathematics permitting the invariance of the *constant* speed c (conventional numbers, fields, spaces, Lie algebras, Lie groups, etc.) is insufficient for the invariance of the *locally varying* speed C, thus mandating the construction of a new mathematics before even addressing the indicated physical problem, as outlined below.

Due to the lack of achievement of the invariance of C, Lorentz pioneering work [1e] was forgotten. The majestic achievements of special relativity in vacuum then created a dark shadow over the physics of the entire 20-th century due to the known suppression of the continuation of Lorentz studies [1e] for physical conditions beyond those of original conception by Lorentz and the other Founders (Section 1).

The author always studies the originators of basic theories and reads with reservations presentations by their followers because often manipulated for personal preferences. Hence, contrary to popular trends during the 20-th century, the author essentially dedicated his research life to the solution of Lorentz problem [1e] with research initiated in 1967 as part of his Ph. D. in physics [3a,3b].

The author view was and remains that an effective way to search for the invariance of $C = c/n$ (where n at this point can be considered a smooth but otherwise arbitrary non-null function) is to construct a broadening of the basic theory leading to the invariance of c, Lie's theory with product $AB - BC$ where A, B can be matrices, vector fields, operators, etc. Therefore, the author proposed since these early papers of 1967 the first generalization of Lie's theory in physics record, that of Lie-admissible type with product $pAB-qBA$ were $p, q, p{\pm}q$ are non-null scalars. This generalization was resurrected later on by Biedenharn and Macfairlane in 1986 as "q-deformations" $AB-qBA$ without a quotation of their origination [1a,1b] (fully known to Biedenharn since we applied for a DOE grant together on Lie-admissibility), resulting in the subsequent well known plethora of papers in the field (see [3j] for comprehensive literature and historical notes).

Ironically, the author had abandoned the study of his (p, q)-deformations by 1986 because of truly catastrophic mathematical and physical inconsistencies that emerged since 1967 (see Refs. [11] and details below). it essentially emerged that the achievement of a true invariance of $C = c/n$ intended as that for c (i.e., have the same numerical predictions under the same conditions at different times) was not allowed by a "mixture" comprising, on one side, the generalization of Lie algebras and groups and, on the other side, its treatment via the conventional mathematics of Lie's theory (conventional numbers, fields, spaces, functional analysis, etc.

It was painfully learned in this way that *to achieve invariance, every generalization of Lie's theory requires its own mathematics.* The understanding of this statement requires the knowledge that no truly new physical theory can be claimed without a truly new mathematics, and no mathematics can be claimed to be truly new without new numbers.

In summary, the greatest majority of the research by the author, a theoretical physicist, was conducted during the past forty years to search search *the appropriate mathematics* leading to the desired invariance. Physical research required proportionate minute efforts since rigidly set by the selected mathematics. An illustration of the complexity of the problem is indicated by the fact that full mathematical maturity was only reached thirty years following the initiations of the studies, with mathematical memoir [3e] for which the *Rendiconti Circolo Matematico di Palermo* dedicated an entire issue (a second article was that by Kadeisvili [8h] and others).

These studies resulted in a number of contributions only partially listed in [3-12] (see the 90 pages long General Bibliography of monograph [6i]). Comprehensive presentations are available in monographs [6] by the author and monographs [12] by independent researchers. The main result is a structural generalization of special relativity today known under the name of *Santilli isospecial relativity* [6,12]. It is evidently impossible to review this long journey and we have to restrict in this note to the main lines.

The problem under consideration is the identification of the relativity applicable within physical media. This problem can be addressed in a variety of ways. The solution here adopted for a study of dark matter in reversible conditions is the broadening of special relativity (called *lifting*) verifying the conditions of:

1) Representing arbitrary maximal causal speeds in the universe irrespective of whether bigger, equal or smaller than the speed of light in vacuum c;

2) Achieving an invariant formulation of arbitrary speeds similar to that of special relativity for the speed of light in vacuum, in the indicated sense of admitting the same numerical values under the same conditions at different times; and

3) Reaching a covering of special relativity, in the sense of admitting the latter uniquely and unambiguously whenever the treatment returns to be in vacuum.

Condition 1 requires the embedding of Lorentz speed (2.1) in a corresponding signature $(+,+,+,-)$-preserving lifting of the Minkowskian spacetime of the type

$$\eta = Diag.(1,1,1,c^2) \rightarrow \hat{\eta} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, c^2/n_4^2) =$$

$$= \hat{T}(t,r,v,\mu,\tau,...) \times \eta = (\hat{T}_\rho^\alpha \times \eta_{\alpha\sigma}),$$

$$C = c/n_4, \quad \hat{T} = \hat{T}^\dagger > 0, \quad n_\rho > 0, \quad \rho = 1,2,3,4. \tag{2.2}$$

where: $n_4 = n$ and $C = c/n_4$ can now be arbitrarily bigger or smaller than c (during Lorentz's times superluminal speeds were unknown) and $1/n_4$ can be interpreted as a representation of the *density* of the medium considered normalized to the value $1/n_4 = 1$ for the vacuum; $n_k, k = 1,2,3$, represent the *extended, nonspherical and deformable shape* of the medium considered, here restricted for simplicity to be a spheroidal ellipsoid due to the diagonal character of $\hat{\eta}$ assumed for simplicity, with normalization to the perfect sphere in vacuum with semiaxes $n_k = 1,1,2,3$ (broader models are trivially produced via nondiagonal realizations of $\hat{\eta}$); the lifting of all elements of the Minkowskian metric is derived from coordinate transformations because it is sufficient to lift only one elements of η for the generalization of all others to follow from coordinate transforms; and the 4×4 matrix $\hat{T}$ is assumed to have (a sufficiently smooth but otherwise) arbitrary dependence on local variables such as time t, coordinates r, velocities v, density μ (not renormalized to the value 1 for the vacuum), temperature τ, etc., as necessary for any serious study of interior dynamical problems. In any case, as we shall see, isorelativity leaves completely unrestricted the functional dependence of the lifted metric.

The verification of condition 2 required rather laborious studies on the *isotopies (i.e., axiom preserving liftings) of Lie's theory,* today known as the *Lie-Santilli isotheory* initiated by the author in Refs. [3c] of 1978 while the author was at Harvard University under DOE support, and continued in numerous works (see monographs [6] by the author and [12] by independent colleagues). We are here referring to an axiom preserving generalization of universal enveloping associative algebras, Lie algebras, Lie groups, and the representation theory generated by the broadening of the basic unit of the Minkowski spacetime into the topology-preserving *inverse* of the matrix $\hat{T}$, with related associative and Lie-Santilli product

$$I = Diag.(1,1,1,1) > 0 \rightarrow \hat{I} = 1/\hat{T} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) > 0. \tag{2.3a}$$

$$A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B, I \times A = A \times I \equiv A, \hat{I} \hat{\times} A = A \hat{\times} \hat{I} \equiv A, \tag{2.3b}$$

$$X_i \times X_j - X_j \times X_i = C_{ijk} \times X_k \rightarrow X_i \hat{\times} X_j - X_j \hat{\times} X_i = C'_{ijk} \hat{\times} X_k. \tag{2.3c}$$

These studies resulted in a new mathematics, today known as *Santilli's isomathematics,* comprising: new numbers, the isonumbers with a arbitrary unit [3d]; new metric and vector spaces [3e]; new algebras, groups and symmetries [3e]; new functional analysis [7,8], etc. Most important has been the achievement of the novel *isotopology* by the Greek mathematicians D. S, Sourlas and the (late) Gr. Tsagas [12d] and the Spanish mathematicians M. Falcon Ganfornina and J. Nunez Valdes [12g].

In turn, the new isotopology allowed the first structural generalization of Newton's equations since Newton's times, that for extended, nonspherical and deformable bodies [3e] and consequential isotopies of classical and operator mechanics known under the name of *hadronic mechanics* [6]. Note that any attempts at the indicated enlargements via conventional mathematics would be catastrophically inconsistent, trivially, due to the Euclidean topology underlying Newton's equation, and its strictly local differential character, that is, solely capable of representing point particles.

The Lie-isotopic generalization of Lie's theory with product $A \times \hat{T} \times B - B \times \hat{T} \times A$ was selected over the broader Lie-admissible version $A \times \hat{R} \times B - B \times \hat{S} \times A$ for the construction of isorelativity because the former allows the preservation of conventional total conservation laws (the product being totally antisymmetric), while the latter does not, thus being valid solely for open nonconservative system. The latter feature alone is sufficient to invalidate a large number of papers on q-deformations not following the lines herein considered.

Following the achievement of the applicable formulation of Lie's theory, Santilli conducted systematic studies for the isotopic lifting of *all* structural aspects of special relativity, including: the lifting $\hat{O}3$ of the rotational symmetry [4a]; the lifting $\hat{SU}(2)$ of the spin symmetry [4b]; the lifting $\hat{O}(3.1)$ of the Lorentz symmetry in its classical [4d] and operator form [4e]; the lifting $\hat{P}(3.1)$ of the Poincaré symmetry [4e]; the lifting $\hat{SL}(2.C) \times \hat{\tau}(3.1)$ of the spinorial covering of the Poincaré symmetry [4f,4g[; the lifting $\hat{M}(3.1)$ of the Minkowskian geometry [4h]; and the applications of the preceding liftings to the EPR argument, local realism and all that [4g] (see [4i] for a review).

The isotopic (that is, axiom-preserving) character of the above liftings is established by the fact that, due to the positive-definiteness of $\hat{I}$, the structure constants of the lifting remain the same $C'_{ijk} = C_{ijk}$ (for the "regular" isotopies with exceptions), in which case all preceding generalized symmetries are locally isomorphic to the original ones, $\hat{O}(3) \approx O(3), \hat{SU}(2) \approx SU(2)$, etc. We generally have broader *realizations* of said symmetries (generally of nonlinear, nonlocal and nonpotential character), but not an alteration of the basic axioms.

Besides the above formal treatment of the isotopic liftings of all main aspects of special relativity, Santilli [5a] identified a method for their simple construction consisting in identifying the lifting of the basic Minkowskian unit with a 4×4 noncanonical (at the classical level) or nonunitary transform (at the operator level),

$$I \to \hat{I} = U \times I \times U^{\dagger}, \quad U = Diag.(n_1, n_2, n_3, n_4), \tag{2.4}$$

and then applying the resulting transform to the *totality* of the mathematical and physical quantities of special relativity and *all* their operations with no exception known to the author, including the isotopies of numbers, products, metric spaces, topologies, algebras, groups, symmetries, geometries, etc., with classical liftings of the type

$$a \to \hat{a} = U \times a \times b = a \times (U \times U^{\dagger}) = a \times \hat{I}, \quad a \in \mathcal{C}, \quad \hat{A} \in \hat{\mathcal{C}}, \tag{2.5a}$$

$$a \times b \to \hat{a} \hat{\times} \hat{b} = \hat{a} \times \hat{T} \times \hat{b}, \tag{2.5b}$$

$$e^X \to U(e^X)U^{\dagger} = \hat{e}^X = (e^{X \times \hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}X}), \tag{2.5c}$$

$$A \times B - B \times A \to \hat{A} \hat{\times} \hat{B} - \hat{B} \hat{\times} \hat{A}, \ \ etc. \tag{2.5c}$$

where: $\mathcal{C}$ is the ordinary field of complex numbers ($\mathcal{R}$ the field of real numbers) and $\hat{\mathcal{C}}$ is ($\hat{\mathcal{R}}$) Santilli isofield of isocomplex (isoreal) numbers [3d]; the conventional (associative) product $\times$ is adopted hereon for all operation pertaining to special relativity and its isotopic image (the isoproduct) $\hat{\times}$ is adopted hereon for all formulations of isorelativity; quantities A, B, etc. are formulated on conventional spaces over conventional fields and their isotopic images $]\hat{A} = U \times A \times U^{\dagger}, \hat{B} = U \times B \times U^{\dagger}$, etc. are formulated on Santilli isospaces over isofields; and the corresponding operator liftings of quantum into *hadronic mechanics* [3c,5a,6g,6h,6i] are here ignored for simplicity.

The nontriviality of the isotopies should be indicated to prevent major misrepresentations rather natural in a superficial glancing of these studies. It can be best indicated by recalling that the $4-4$-matrix $\hat{T}$ has a generally nonlinear and integrodifferential dependence on the variable. The appearance of $\hat{T}$ in the *exponentiation,* Eq. (2.5c), that is, in the group structure, shows that the isotopies transform the linear, local-differential and Hamiltonian Lie theory into the most general possible, axiom-preserving nonlinear, nonlocal-integral and non-Hamiltonian form.

The necessity of lifting the entire formalism of special relativity without exception should be stressed because catastrophic inconsistencies follow in the event of partial liftings. As an illustration, it is possible to lift the entire Lie theory without lifting the basic numbers and fields into Santilli's isonumbers and isofields. In this case the resulting theory appears to be consistent, while in reality it verifies the so-called *theorems of catastrophic inconsistencies of noncanonical or nonunitary theories* [3j,11]. This is due to the fact that the lifted theory is noncanonical at the classical; level and nonunitary at the operator level, namely, characterizes transformations that, by their very definition, do *not* conserve the basic unit.

The lack of invariance of the basic unit causes catastrophic mathematical inconsistencies because of the lack of preservation of the basic numerical field at a later times, with consequential collapse of metric spaces, topologies, symmetries, etc. On physical grounds, noncanonical-nonunitary theories cannot possibly preserve numerical values over time, besides having additional catastrophic consequences, such as the loss of observability (known as the *Lopez lemma* [11b,11c]), causality, and other basic physical laws [11i].

Santilli's isotopies bypass these inconsistencies because they reconstruct canonicity and unitarity on isospaces over isofields, called *isocanonicity and isounitarity* in which case invariance is recovered in full as in the cases [3e]

$$U = \hat{U} \times \hat{T}^{1/2} \tag{2.6a}$$

$$U \times U^\dagger \neq I \rightarrow \hat{U} \hat{\times} U^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \tag{2.6b}$$

$$\hat{I} \rightarrow \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}, \tag{2.6c}$$

$$\hat{A} \hat{\times} \hat{B} \rightarrow \hat{U} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} U^\dagger = \hat{A}' \hat{\times}' \hat{B}' \equiv \hat{A}' \hat{\times} \hat{B}', \quad etc., \tag{2.6d}$$

where the reader should note the *preservation of the numerical value* of the unit $\hat{I} \rightarrow \hat{I}' \equiv \hat{I}$ and of the product $\times \rightarrow \times' \equiv \times$ exactly as it happens for special relativity under canonical or unitary transforms.

Following Santilli's research [3], a number of generalizations of special relativity have appeared in the subsequent literature but they are fundamentally inconsistent because of the lack of invariance as in Eqs. (2.6), and consequential activation of the inconsistency theorems [3j,11]], as the reader is encouraged to verify. To prevent research that cannot possibly resist the test of time, the use of these inconsistent generalizations is discouraged.

We cannot possibly review Santilli's isorelativity in the necessary details to avoid a prohibitive length, and have to refer the interested reader to the specialized literature [6,12]. We merely recall for minimal selfsufficiency of this note the *general isoinvariant of the Minkowski-Santilli isospace and isogeometry* defined on the *Minkowski-Santilli isospace* $\hat{M}(\hat{x}, \hat{\times}, \hat{I})$ (see monographs [6g,6h,6i] for a detailed presentation and references)

$$(\hat{x}-\hat{y})^{\hat{2}} = (\hat{x}-\hat{y})^{\mu}\hat{\times}\hat{N}_{\mu\nu}\hat{\times}(\hat{x}-\hat{y})^{\nu} = [(x-y)^{\mu}\times\hat{\eta}_{\mu\nu}\times(x-y)^{\nu}]\times\hat{I} =$$

$$= [(x^1-y^1)\times\hat{T}_{11}\times(x^1-y^1)+(x^2-y^2)\times\hat{T}_{22}\times(x^2-y^2)+$$
$$+(x^3-y^3)\times\hat{T}_{33}\times(x^3-y^3)-(x^4-y^4)\times\hat{T}_{44}\times(x^4-y^4)\times\hat{I}. \tag{2.7}$$

Representative examp[les of the explicit form of the isosymmetry transformations leaving invariant the above isoline element are given by (see again [6g,6h,6i] for details):

1) Isorotations $\hat{)}(3)$ [4a], here expressed for the (x,y)–plane were we ignore hereon the factorization of $\hat{I}$ for simplicity (see monograph [6h] for the general isorotations in three dimensional isospaces)

$$x' = x\times\cos(\hat{T}_{11}^{\frac{1}{2}}\times\hat{T}_{22}^{\frac{1}{2}}\times\theta_3) - y\times\hat{T}_{11}^{-\frac{1}{2}}\times\hat{T}_{22}^{\frac{1}{2}}\times\sin(\hat{T}_{11}^{\frac{1}{2}}\times\hat{T}_{22}^{\frac{1}{2}}\times\theta_3),$$

$$y' = x\times\hat{T}_{11}^{\frac{1}{2}}\times\hat{T}_{22}^{-\frac{1}{2}}\times\sin(\hat{T}_{11}^{\frac{1}{2}}\times\hat{T}_{22}^{\frac{1}{2}}\times\theta_3) + y\times\cos(\hat{T}_{11}^{\frac{1}{2}}\times\hat{T}_{22}^{\frac{1}{2}}\times\theta_3), \tag{2.8}$$

Isorotations (2.8) leave invariant all ellipsoidal deformations of the sphere, as the reader is encouraged to verify. The local isomorphism between $\hat{O}(3)$ and $O(3)$ then confirms the perfect spheridicity of the "isosphere", namely, conventional ellipsoids when reformulated on isospace over isofields because the deformation of the semiaxes of the sphere $1_k \rightarrow 1/n_k^2$ is compensated by the b inverse lifting of the corresponding units $1_k \rightarrow n_k^2$ [6h].

2) Santilli-Lorentz isotransformations $\hat{O}(3.1)$ [4c], which are characterized by the isorotations and the *isoboosts* here expressed for simplicity in the $(3,4)$–plane (see, again, [6h] for the general case)

$$x^{3\prime} = x^3\times\sinh(\hat{T}_{33}^{\frac{1}{2}}\times\hat{T}_{44}^{\frac{1}{2}}\times v) - x^4\times\hat{T}_{33}^{-\frac{1}{2}}\times\hat{T}_{44}^{\frac{1}{2}}\times\cosh(\hat{T}_{33}^{\frac{1}{2}}\times\hat{T}_{44}\times v) =$$

$$= \tilde{\gamma}\times(x^3-\hat{T}_{33}^{-\frac{1}{2}}\times\hat{T}_{44}^{\frac{1}{2}}\times\hat{\beta}\times x^4)$$

$$x^{4\prime} = -x^3\times\hat{T}_{33}\times c_0^{-1}\times\hat{T}_{44}^{-\frac{1}{2}}\times\sinh(\hat{T}_{33}^{\frac{1}{2}}\times\hat{T}_{44}\times v) + x^4\times\cosh(\hat{T}_{33}^{\frac{1}{2}}\times\hat{T}_{44}^{\frac{1}{2}}\times v) =$$

$$= \tilde{\gamma}\times(x^4-\hat{T}_{33}^{\frac{1}{2}}\times\hat{T}_{44}^{-\frac{1}{2}}\times\tilde{\beta}\times x^3),$$

$$\tilde{\beta} = v_k\times\hat{T}_{44}^{\frac{1}{2}}/c\times\hat{T}_{44}^{\frac{1}{2}}, \tilde{\gamma} = (1-\tilde{\beta}^2)^{-\frac{1}{2}} \tag{2.9}$$

Note that the above isotransforms are formally similar to the Lorentz transforms, as expected from their isotopic character.

Note also that, when considered in our spacetime (i.e., when referred to our unit), the above isotransform characterize a deformed light cone as an evident necessary condition to represent arbitrary speeds. However the *isolight cone,* namely the preceding deformed cones formulated on the Minkowski-Santilli isospace over isofield is a perfect light cone. This implies the important *reconstruction on* $\hat{M}(3.1)$ *over* $\hat{\mathcal{R}}$ *of the speed of light* c_o *in vacuum as the maximal causal speed within physical media.* The understanding is that calculation for direct applications occur with the deformed light cone in our spacetime, the isolight cone being needed for axiomatic consistency.

3) Isotranslations $\hat{\mathcal{T}}$ [4e], which can be written

$$x' = (\hat{e}^{i\times\hat{p}\times a})\hat{\times}\hat{x} = [x + a\times A(x,v,d,...)]\times\hat{I}, \hat{p}' = (\hat{e}^{i\times\hat{p}\times a})\hat{\times}\hat{p} = \hat{p},$$

$$A_\mu = \hat{T}^{1/2}_{\mu\mu} + a^\alpha \times [\hat{T}^{1/2}_{\mu\mu}, \hat{p}_\alpha]/1! + ... \tag{5.12}$$

and they are nonlinear, as expected and desired.

4) Isoinversions [4e], which are given by

$$\hat{\pi}\hat{\times}x = \pi\times x = (-r, x^4), \hat{\tau}\hat{\times}x = \tau\times x = (r, -x^4) \tag{2.10}$$

where $\hat{\pi} = \pi\times\hat{I}, \hat{\tau} = \tau\times\hat{I}$, and π, τ are the conventional inversion operators.

5) Isoselfscalar transforms [4e], which are characterized by invariances (3.3)-(3.4), i.e.,

$$\hat{I}\rightarrow\hat{I}' = \rho^2\times\hat{I}, \hat{\eta}\rightarrow\hat{\eta}' = \rho^{-2}\times\hat{\eta}, \tag{2.11}$$

where ρ is the parameter characterizing the novel 11-th dimension.

A few comments are in order for the benefit of readers not yet knowledgeable of the new isorelativity. The studies under consideration here have discovered that, contrary to a popular belief throughout the 20-th century, *the conventional Poincaré symmetry* $P(3.1)$ *is eleven dimensional and not ten dimensional as stated in the technical literature.* Consequently the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ is eleven dimensional. This is due to the discovery of the new symmetry of the conventional Minkowskian line element here expressed for simplicity in terms of scalars S, S' [3c]

$$x^2 = (x^\mu\times\eta_{\mu\nu}\times x^\nu)\times I \equiv [x^\mu\times(S\times\eta_{\mu\nu})\times x^\nu]\times(S^{-1}\times I) \equiv (x^\mu\times\hat{\eta}_{\mu\nu}\times x^\nu)\times\hat{I} \equiv$$

$$\equiv [x^\mu \times (S'\hat{\eta}_{\mu\nu}) \times x^\nu] \times (S'{-}1 \times \hat{I}) \equiv (x^\mu \times \hat{\eta}'_{\mu\nu} \times x^\nu) \times \hat{I}'. \quad (2.12)$$

The above new symmetry escaped the attention of colleagues during the 20-th century because its identification required the prior discovery of *new numbers,* Santilli's isonumbers with arbitrary unit $\hat{I}$.

Rather than being trivial, the discovery of the 11-th dimension of the conventional Poincaré symmetry permitted the first and only achievement known to the author of an *axiomatically consistent grand unification of electroweak and gravitational interactions for both matter and antimatter* [5g,5h]. Antimatter was excluded by all preceding attempts, resulting in catastrophic internal inconsistencies (since electroweak interactions do include antimatter while conventional gravitation cannot. as shown in Section 1), plus catastrophic inconsistencies caused by the lack of a consistent quantum formulation of gravity (electroweak interactions are consistently quantized while gravitation cannot) and numerous other inconsistencies. As a matter of fact, *isogranduniﬁcation* [5g,5h] explained the reason for the failure of all preceding attempts at grandunifications since Einstein's times as resting in the absence of the technically and axiomatically appropriate symmetry, the yet unknown 11-th symmetry.

The literature of the 20-th century is populated by statements of spacetime and internal symmetries as being broken in nature. All these statement are technically erroneous because they do not clarify that the breaking occurs due to the use of the simplest conceivable mathematics, that based on numbers dating back to biblical times. In fact, symmetries claimed to be broken in the literature have been reconstructed as exact via the use of the Lie-Santilli isosymmetry beginning in 1978 [3c], the apparent unwillingness by researchers to study this covering theory (with numerous exceptions [7-12]) causing the inability of their papers to pass the test of time.

In fact, Eqs. (2,8) are a visual proof of the reconstruction of the exact rotational symmetry for all ellipsoidal deformations of the sphere, since for any such ellipsoid there exist a related Santilli isounit for which the image of the ellipsoid on isospace over isofield is the perfect sphere (the isosphere). Similarly, Eqs. 2.9) reconstruct as exact on isospaces over isofields all (signature-preserving) breakings of the Lorentz symmetry, *e.g.,* breakings characterized by speeds of light different than c, because the isotopies reconstruct c as the maximal causal speed on said isospaces over isofields. Ref. [4g] has established the exact character of the $SU(2)$-isospin symmetry in nuclear physics, in which field the symmetry is believed to be broken by weak interactions. Ref. [6h] has shown the reconstruction of the exact character of parity on isospaces over isofields. All these reconstructions are done via the simple embedding of all symmetry breaking terms into Santilli isounit, and then the reconstruction of the symmetry with resect to that isounit. Interested readers can reconstruct as exact any symmetry believed to be broken via the same procedure.

Isorelativity is the set of all preceding formulations, plus the isotopies of the basic axioms of special relativity reviewed in a systematic way in monographs [6e,6f] of 1991, here expressed for the simpler case in $(3,1)$-space with $n_3 \neq n_4$ as a necessary condition to represent the *anisotropy and inhomogeneity of physical media in real life* (see Refs. [6h] for a detailed presentation of the general case):

ISOPOSTULATE I: The maximal causal speed within physical media is given by

$$\hat{V}_{Max} = c \times n_3/n_4. \tag{2.13}$$

ISOPOSTULATE II: The addition of speeds within physical media follows the isotopic law

$$\hat{v}_{Tot} = (u+v)/(1 + u_3 \times n_3^{-2}. \times v_3/c \times n_4^{-2} \times c). \tag{2.14}$$

ISOPOSTULATE III: The dilation of time and the space contraction within p hy si cal media follow the isotopic laws

$$\hat{t}' = \hat{\gamma} \times t, \quad \hat{L}' = \hat{\gamma}^{-1} \times L, \tag{3.15}$$

$$\hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2}, \hat{\beta} = v_3/V_{Max} \leq 1. \tag{2.16}$$

ISOPOSTULATE IV: The isodoppler law for within physical media is given by the expression (for the simple case of null aberration)

$$\hat{\omega}' = \hat{\gamma}^{-1} \times \hat{\omega}. \tag{2.17}$$

ISOPOSTULATE V: The mass-energy equivalence within physical media follows the isotopic law

$$\hat{E} = m \times C^2 = m \times c^2/n_4^2. \tag{2.18}$$

A few comments are now in order for the noninitiated reader. Firstly, we should indicate that, rather than violating Einsteinian axioms as often results from a superficial glancing of the literature, all efforts of Refs. [3-6] are aimed at *preserving* Einsteinian axioms and merely looking for their *broader realization.* In fact, *isorelativity and special relativity coincide at the abstract level* to such an extend that they can be both represented with the same equations in which symbols are then subjected to different interpretations. Consequently, *isorelativity de facto provides the broadening of the validity of Einsteinian axioms, from their original area of empty space, to dramatically broader conditions existing in the universe.*

In fact, *the isorelativistic realization of Einsteinian axioms have been proved to be directly universal* in the sense that it contains as particular cases all infinitely possible signature-preserving generalizations of the Minkowski spacetime for matter in reversible conditions (universality) directly in the frame of the experimenter, that is, without any need of coordinate transformations (direct universality).

The above features are crucial to prevent the possible superficial perception that the representation of dark matter proposed in the next section "violates" Einsteinian axioms, a perception technically erroneous and disrespectful toward the memory of Albert Einstein.

Secondly, the reader should be aware that, even though continued to be ignored for decades to the sole detriment of the uninformed, the above isoaxioms have been subjected to verifications in particle physics, nuclear physics, superconductivity, chemistry and astrophysics. We cannot possibly review here these verifications and have to refer the interested reader to monographs [6,12] and quoted literature. As an example, isorelativity has provided an elementary, but exact representation of the large numerical differences existing in cosmological redshifts between a galaxy and a quasar when proved to be physically connected according to gamma spectroscopy and other evidence. The large difference is merely due to the fact that light travels the immense chromospheres of quasars at much reduced speed, thus existing said chromospheres at a value of the Doppler-Santilli isoredshift much bigger than the corresponding one of light emitted from the connected galaxy. This has been an important verification of Santilli isorelativity worked out by the Italian physicist R. Mignani [10j].

Thirdly, the reader should be aware that in isorelativity *the speed of light is not, in general, the maximal causal speed,* with the sole exception of motion in vacuum (where the two speeds trivially coincide). This new axiomatic vista resulted to be necessary, first of all, to have a maximal causal speed applicable also to opaque media, but also for the resolution of the inconsistencies of special relativity when applied within water (Section 1).

Recall that water is isotropic and homogeneous to a considerable extent, thus demanding that $n_3 \equiv n_4$, in which case the maximal causal speed in water is c from Eq. (2.13). This is necessary to verify causality, namely, to avoid that ordinary electron travel in water faster than the maximal causal speed. The isorelativistic sum of speeds, Eq. 2.14, is then valid for the maximal causal speed, and not for a locally varying speed such as that of light.

Next, the noninitiated reader should be aware that the isotopies of the Riemannian formulation of gravity are inessential and, if attempted, are afflicted by the same mathematical and physical inconsistencies of the original formulation [11j]. This is due to the fact that *isorelativity as formulated above is fully inclusive of gravitation,* thus explaining the single formulation "isorelativity", rather than having the two formulations "isospecial" and "isogeneral" relativities. As a matter of fact, a primary function of isorelativity is that of providing a *geometric unifications of both special and general relativity* [4i].

This important feature is due to the fact that Santilli's isotopies leave unaffected the functional dependence of the isounit. Consequently, *all Riemannian line elements are a particular case of the general invariant* (2.7) under the following isominkowskian reinterpretation of any given Riemannian metric with signature $(+,+,+,-)$ [5c]

$$g(x) = \hat{T}(x) \times \eta, \quad \hat{I} = 1/\hat{T}. \tag{2.19}$$

and then the use of the Minkowski-Santilli isogeometry [4i] that admits (under isotopic lifting) the totality of the machinery of the Riemannian geometry, such as covariant derivatives, Christoffel symbols, etc., trivially, because the isominkowskian metric $\hat{\eta}(x)$ does carry a dependence on spacetime coordinates. This implies the identical reformulation of the Einstein-Hilbert gravitational field equations [2b-2c] on isominkowski spaces over isofields.

The implications for gravitation of its isominkowskian reformulation are dramatic and cannot be reviewed in detail here. We merely mention that the reformulation is centered in *the Poincaré-Santilli isosymmetry as the universal symmetry, rather than covariance, of all gravitational models* [4]. In turn, such an invariance avoids the theorems of catastrophic inconsistencies of conventional gravitation on a curved space originating from its noncanonical character [3j,11h,11i].

Yet another important implication is that *the isominkowskian reformulation of gravity has no curvature,* as necessary for a credible representation of the free fall of bodies along a straight radial line, to avoid the prediction of the double bending of light (since that observed is purely Newtonian in nature and not due to curvature), and other evidence.

This is due to the fact that the Minkowskian geometry has no curvature; isotopies preserve all original axioms; hence, the Minkowski-Santilli isogeometry equally has zero curvature. This feature can be empirically seen from the fact that curvature is contained in the 4×4-matrix $\hat{T}$ in the factorization $g(x) = \hat{T} \times \eta$. Consequently, curvature persist when the above factorization is referred to the conventional unit I of the Riemannian space, but said curvature disappear when referred to the isounit $\hat{I} = 1/\hat{T}$ that is the *inverse* of the "curvature matrix since the structure of the basic invariance is essentially given by metric (isometric) multiplied by the unit (isounit), the latter condition being necessary for the proper characterization of the line element as a scalar (isoscalar). In particular, the bypassing of the theorems of catastrophic inconsistencies of gravitation [11j] is due precisely to the abandonment of curvature.

Another consequence important for dark matter is the achievement, for the first time known to this author, of *an axiomatically consistent operator formulation of gravity,* permitted precisely by the abandonment of curvature, that was the origin of all inconsistencies of quantum gravity. This operator form is directly provided by *relativistic hadronic mechanics* [5a],and is merely given by the embedding of gravity where nobody looked for, in the *unit* of conventional relativistic quantum mechanics. Under this procedure the validity of all conventional quantum axioms remains fully valid, thus guaranteeing the consistency of *operator isogravity* beyond credible doubt. This allows studies such as the *isotopic reformulation of Dirac equation* with the isometric $\hat{\eta} = g(x)$, where $g(x)$ is the conventional Schwartzchild metric [2d], and similar gravitational formulations (see [6h,6i] for brevity).

Note the insistence since the beginning of this section in the restriction of physical media to *reversible* conditions. The lifting of the above formulation to include *irreversibility* has been done along two sequential approaches. The first approach requires the admission of internal irreversibility for matter when considered isolated from the rest of the universe, thus verifying conventional total conservation laws. The second approach deals with irreversibility for open systems verifying general nonconservation laws.

The former case is easily described with the above formulations by merely admitting a time dependence such that

$$\hat{I}(t, ...) \neq \hat{I}(-t, , ...), \tag{2.20}$$

In this case conventional total conservation laws are trivially guaranteed by isorelativity from the fact that the generators of the Poincaré-Santilli isosymmetry coincide with the conventional ones [4].

The extension of isorelativity to irreversibility under open conditions required dramatically more efforts because, after numerous trials and errors, it emerged necessary to construct a *second new mathematics,* today known as *Santilli genomathematics* [3f] so as to embed irreversibility in the very mathematical structure of the theory. This second extension is essentially based on the use of non-Hermitean genounits

$$\hat{I}^{>} \neq ({}^{<}\hat{I})^{\dagger}, \tag{2.21}$$

where the upper symbols $>, <$ represent the arrow of time. The *dual genotopies* of special relativity, one per each direction of time, then yield *Santilli genorelativity* for matter in irreversible conditions we cannot possibly review here (see the recent memoir [3j]). As a result of these studies, thermodynamics can be connected, apparently for the first time, to dynamics as pioneered by the British scientist J. Dunning-Davies [10j].

Finally, the reader may have noted the systematic exclusion of antimatter in this section so far. This is due to the fact that *isorelativity is as catastrophically inconsistent for a classical representation of antimatter as special relativity is.* In fact, isotopies carry along not only the good features of the original theory, but also its physical and/or mathematical inconsistencies since they do not change the axioms.

The inclusion of antimatter required a third laborious effort that lasted for decades. The only consistent map from particles to antiparticles existing in the literature of the 20-th century is charge conjugation, which is an anti-homomorphism, as well known. But charge conjugation can only be applied on Hilbert spaces $\mathcal{H}$ over $\mathcal{C}$, without any classical image, as also well known. An extensive search conducted at the Cantabridgean libraries in the early 1980s (while the author was at the Department of Mathematics at Harvard University) revealed that a mathematics permitting an anti-homomorphic or anti-isomorphic image of relativities (as needed for a classical representation of antimatter) simply did not exist.

As a result, following various trials and errors, the author had to construct a *third new mathematics,* this time anti-isomorphic to conventional mathematics, today known as *Santilli isodual mathematics* [5k-5m,6n] that is characterized by the following *isodual map*

$$\hat{I}(t, r, v, \mu, \tau, ...) \rightarrow \hat{I}^{d}(t^{d}, r^{d}, v^{d}, \mu^{d}, \tau^{d}, ...) = -\hat{I}\dagger(-t^{\dagger}, -r^{\dagger}, -v^{\dagger}, -\mu^{\dagger}, -\tau^{\dagger}, ...), \tag{2.22}$$

applied to the *totality* of the formulations of special relativity (for the treatment of antimatter in vacuum) and isorelativity (for antimatter within physical media). Isodual isorelativity then automatically includes a gravitational treatment of antimatter. The interplay between isorelativity and its isodual predicts *antigravity* as the gravitational repulsion of antimatter in the field of matter and vice versa (see monograph [6n] for detailed studies and proposed experiments).

This resulting *isodual theory of antimatter* has a rather strong experimental verification because the theory trivially verifies all available classical data on antimatter while, at the operator level, isoduality is equivalent to charge conjugation [6n].

The reader should be aware that no systematic astrophysical study of antimatter in the universe has been conducted to date primarily because of the lack of a consistent classical treatment of antimatter at both the relativistic and gravitational levels (Section 1). It is hoped that the resolution of this insufficiency permitted by the above advances will finally allow the initiation of serious studies of antimatter in the universe.

In the final analysis we should not forget that up to half of the universe could be composed of antimatter for a variety of reasons, such as the fact that gravitational repulsion between matter and antimatter galaxies, now quantitatively treatable via isotopies and isodualities, is the most logical explanation not only of the expansion of the universe, but also of its increase in time.

To close this section with a personal comment, the author has noted that a difficulty for a wider propagation of the above studies so far has been a resiliency shown by theoreticians toward the needed new mathematics. Readers experiencing such a resiliency should be reminded that *there cannot be really new physical theories without really new mathematics, and there cannot be really new mathematics without new numbers.*

3. Isorelativistic representation of dark matter

The view submitted by the author in this note is that the apparent insufficiency of visible matter in the universe to explain the dynamics of galaxies and the universe at large is due, at least in part, to the aprioristic assumption of the exact validity of special relativity and the universal constancy of the speed of light c throughout the entire universe.

Without any claim of resolving the entirety of the problem, that would be ascientific due to its complexity, in this note we show, apparently for the first time, that at least a significant part of dark matter (dark antimatter) can be quantitatively explained via Santilli isorelativity (its isodual) without any need of conjecturing invisible or other esoteric entities.

The first suggestion is that *the problem of dark matter and energy should be unified into the sole treatment of dark energy,* with a similar assumption for dark antimatter and energy. At any rate, the terms "dark matter" do not have sufficient physical specifications suitable for a quantitative treatment. This seemingly innocuous assumption has rather deep consequences because it implies that, contrary to a popular belief dating back to Newton's time and fully adopted by Einstein, *gravitation should be described in term of energy and not of mass.*

While leaving epistemological discussions to interested science philosophers, the assumption of treating gravity solely in terms of energy is necessary for a serious description of the universe to avoid dichotomies, insufficiencies or even inconsistencies. As well known, light has no mass, yet it experiences gravity. It is then not proper to represent certain gravitational events in terms of mass and others in terms of energy. The absence of mass for gravitational events such as the bending of light in a gravitational field then leaves the description of gravitation in terms of energy as the sole unified approach consistently applicable throughout the universe.

Once energy is assumed as the origin of gravitation, the next step is the use of the celebrated mass-energy equivalence $E = m \times c^2$. However, this equivalence is certainly valid only under the conditions clearly expressed by Einstein, namely, for point-particles moving in vacuum. For actual, that is, extended, deformable and nonspherical particles with density μ, the above law is no longer necessarily valid and should the replaced with the the covering law of isorelativity, that of Isoaxiom V of the preceding section where $C = c/n_4$ represents the image of the speed c within the interior of the particle considered.

The comparison of Isoaxiom V with the conventional Einsteinian expression for the same particle creates the alternative of either assuming the energy to be the same in the transition from special relativity to isorelativity, in which case mass and speed of light change, or we assume the mass to be the same, in which case energy and speed of light change, and we shall write

$$E = m \times c^2 = M \times C^2, \quad C >> c, M << m, \tag{3.1a}$$

$$m = E/c^2 = \mathcal{E}/\mathcal{C}^{\in}, \quad \mathcal{C} >> \rfloor, \quad \mathcal{E} << \mathcal{E}. \tag{3.1b}$$

Assumption (3.1a) already illustrates a main point for the proposed isorelativistic interpretation of (at least part of) the dark energy. In fact, astrophysical calculations for galaxies and quasars are generally computed in term of the law $E = m \times c^2$ where m is the mass of the universe, which assumption results in the known large energy deficiency needed to represent the behavior of stars, galaxies and quasars.

However, for isorelativity the energy equivalence of a given mass is bigger than the Einsteinian value due to the fact that the value of C inside hyperdense media is much bigger than c [10a,10b,6i], as shown in Eq. (3.1b). Consequently, current estimates on the mass of the universe could yield much bigger energies than those predicted by special relativity.

To provide a numerical indication, we know from the fit of isorelativistic calculations on the Bose-Einstein correlation that the fireball has a density of about $1/n_4 = 1.653 \pm 0.035$ with value $C = 1.653 \times c$, a value that can be assumed as valid in first approximation also for the interior of protons and neutrons, a numerical value confirmed by studies on the synthesis of neutrons from protons and electrons (see monograph [66k] and quoted literature).

When passing to brown holes, gravitational collapse, black holes and other extreme conditions existing in the universe, the value of C can be thousands of times bigger than c. Consequently, isorelativity does indeed appear to permit a representation of (at least part) of the missing energy in the universe without conjecturing invisible entities, and via the sole use of the visible matter. The understanding is that, when referring to a galaxy, the value of C is an average over a variety of astrophysical bodies all having different values of C. We shall then write

$$E_{tot}^{univ} = m_{tot}^{univ} \times C_{aver}^2 >> m_{tot}^{univ} \times c^2, \qquad (3.2)$$

Consequently, dark energy is here assumed to be merely given by the increase of energy over that predicted by special relativity,

$$\Delta E_{darkenergy} = m_{tot}^{univ} \times (C_{aver}^2 - c^2). \qquad (3.3)$$

Hence, the current estimate of the value of the dark energy can be used to provide an estimate of the average value of C for the universe. For instance, assuming that the missing energy is 100-times the Einsteinian value, we get the estimate

$$C \approx 10 \times c, \qquad (3.4)$$

that is a quite reasonable average increase of the speed of light for hyperdense astrophysical objects since in the interior of protons and neutrons we have $C = 1.653 \times c$. Similar average values of C can be obtained for the isorelativistic representation of bigger percentages of the missing energy.

Despite its simplicity and plausibility, the above hypothesis carries a number of rather deep implications as well as open problems. such as:

1) The validity of the isorelativistic representation of dark energy, Eq. (3.3), should be first studied at the level of particle physics. In fact, for a given rest energy, the hypothesis implies a difference in rest masses between the Einsteinian and the isorelativistic treatments, since the former applies only for the point-like abstraction of the particle considered, while the latter includes the extended character and the density of the particle outside descriptive capacities of special relativity. Clearly, the above alternatives have to be resolved at the particle level in favor either of the Einsteinian or of the isorelativistic representation prior to, or jointly with, astrophysical considerations.

2) The special or isorelativistic alternatives should be resolved for the Newtonian treatment of our Solar system. As well known, calculations in the field are based on a known *estimate* (and definitely not a measurement) of the mass of the Sun and planets $m_{sun}, m_{planets}$. The reformulation of Newton's gravitational force in terms of alternative (3.1b) evidently yields the reformulation for the gravitational force between the Sun and a planet

$$F = G \times \frac{E_{sun} \times E_{planet}}{C^2_{sun} \times C^2_{planet} \times r^2}, \tag{3.5}$$

where G is the usual constant.

Despite its seemingly trivial character, the above reformulation may have rather subtle implications for various aspects of our solar system requiring specific investigations for a confirmation or a denial of the isorelativistic representation of gravity.

3) It is an easy additional prediction that no serious study of dark energy can be claimed without a systematic study of the antimatter component in our universe. Until now, astronomers have looked at a distant galaxy and automatically assume that it is made up of matter because of the complete lack of theories capable of distinguishing between matter and antimatter as stressed in Section 1. This serious imbalance between matter and antimatter in the physics of the 20-th century has been resolved by the isodual theory of antimatter [6n]. It is, therefore, hoped that serious studies on antimatter in the universe are initiated, *e.g.,* via the prediction that *antimatter appears to emit a new light* different than that of matter, besides being predicted to have gravitational repulsion in the field of matter and other aspects verifiable with astrophysical measurements. Jointly with these aspects, there is then the need to initiate the study of the energy of "dark antimatter" that can be done via isoduality.

All in all, we can say that the isorelativistic representation of the energy of dark matter and its isodual image for dark antimatter should deserve additional studies among the variety of existing attempts [1i], with the understanding that, in the event correct, the proposed representations create a number of intriguing open problems that, rather than being a drawback, are a direct indication of novelty when seen by young minds of all ages.

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